MATHEMATICAL REPRESENTATIONS BASED ON FIELD-DEPENDENT AND FIELD-INDEPENDENT COGNITIVE STYLES

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ABSTRACT

Mathematical representation ability is essential for students, but many students still need to be able to solve mathematical problems with various mathematical representation abilities. This research is narrative research with a qualitative approach to describe the problems of students' mathematical representation abilities in terms of field-dependent and field-independent cognitive styles on the set topic. A total of 4 students, based on the GEFT test and the results of the subject teacher's directions, tended to field-dependent and field-independent cognitive styles. The instruments used in this study were the GEFT test, questions of representation ability, and interview guidelines. The results of the data analysis show that the mathematical representation abilities of students with a field-dependent cognitive style can be represented with several models. In contrast, the mathematical representation abilities of students with a field-independent cognitive style can represent all indicators. Based on these conditions, the teacher can plan a differentiated lesson in the learning process by paying attention to the differences in students' cognitive styles.

Keywords: Cognitive Style, Mathematical Representation, Set.

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1. INTRODUCTION

Representational ability is one of the student abilities used to help students understand ideas in mathematics (Kwon & Capraro, 2021). Students need mathematical representation abilities to understand mathematics and to construct abstract knowledge into concrete knowledge through logical thinking (Goldin, 2014). Mathematical representation ability is the ability to express mathematics through symbols, pictures or graphs, numbers, and words to solve mathematical problems (Ratumanan et al., 2022). Students are said to
have good mathematical representation skills when using various models to solve mathematical problems.

Mathematical representation ability can be described by four categories of representation which are identified as verbal, numerical, pictorial or pictorial, and algebraic (Nurrahmawati et al., 2021). Students usually use verbal representations to state the problem at the beginning and provide a final interpretation of the answers found in solving the problem. Numerical representation is used to represent the numbers at the solution. Pictorial or image representation is the ability to solve problems with pictures, graphs, or diagrams. Algebraic representation is an expression that is expressed through an example with variables to model solving mathematical problems. Indicators in this study, the ability of representation can be seen from 1) the ability to present answers through pictures or diagrams, 2) present answers through mathematical symbols, 3) present answers through numbers, 4) present answers through verbal or words (Friedlander & Tabach, 2001; Nadira et al., 2023).

The set is one material that shows various kinds of mathematical representations (Sari & Sutirna, 2022). The ability of image representation can be shown through Venn diagrams, symbol representation can be shown through set notation, numerical representation can be shown in set operations, and verbal representation can be shown through words explaining the set situation. However, several studies show that many students still need help using various mathematical representations to solve problems about sets (Wati et al., 2019; Wulandari et al., 2019). This research shows the need for teachers to develop students' mathematical representation skills so that students can solve mathematical problems appropriately.

Each student will have different ways and patterns of thinking when expressing a mathematical understanding of a mathematical problem (Tiew et al., 2023), so each student will have different representation abilities. One factor that differentiates students when presenting the results of answers to math problems is the cognitive style of students (Sellah et al., 2017; Spector, 2012). Cognitive style is a person's habit, a way of thinking that is common or preferred to understand information, process information, and apply information (Hayes & Allinson, 1994). Cognitive style is a person's thinking ability to process, store and use to respond to information (Bassey et al., 2009; Ratuanik, 2018). Cognitive style is a variable that teachers must pay attention to in designing learning because cognitive style can influence the way students learn and the way students interact with teachers in the classroom (Bintoro et al., 2022).

This research centers on the examination of mathematical representation proficiency with a focus on field-dependent (FD) and field-independent (FI) cognitive styles. The impact of FI and FD becomes evident in the manner in which students approach mathematical problem-solving, thereby influencing their overall mathematical representation capabilities(Septian et al., 2020). FD characterizes a cognitive style wherein a student's thought processes are notably susceptible to external influences. Conversely, FI reflects a cognitive style in which a student adeptly processes information without being swayed by external factors (Witkin et al., 1977). By discerning students' cognitive styles, it is anticipated that educators can gain insights into their initial competencies, enabling them to tailor instructional content in alignment with students' cognitive preferences before disseminating the material. This approach ensures that learning objectives are effectively communicated to students in accordance with their cognitive styles (Guisande et al., 2007).

So far, much research has been done on mathematical representation abilities in terms of cognitive style (Himmah & Rahaju, 2021; Khairunnisa & Masrukan, 2020). However, there has yet to be an explanation of the characteristics of mathematical representation abilities in students' FI and FD cognitive styles in solving mathematical problems on set. So
the purpose of this study was to determine the characteristics of mathematical representation ability in terms of students' cognitive style. Along with implementing a new curriculum in Indonesia called Merdeka Curriculum, the Integrated Bilingual Middle School was chosen as the driving school that implemented the independent curriculum. One of the learning approaches to the independent curriculum is differentiation learning. Differentiated learning is a learning approach that the teacher carries out by adjusting learning activities based on each student's learning abilities, interests, and student profile (Tomlinson, 2001). So through differences in student profiles seen from cognitive styles, teachers can provide appropriate learning services to develop students' mathematical representation abilities fully.

2. METHOD

This narrative research uses a qualitative approach (Creswell, 2008). This study aims to describe the characteristics of mathematical representation abilities in terms of students' cognitive style on set material. This research was conducted at the Krian Integrated Bilingual Middle School, Sidoarjo, on 3-30 May 2023. The data collection techniques in this study were cognitive style tests, representation ability tests, and interviews. The research instrument used to collect data about students' cognitive styles uses the Group Embedded Figure Test (GEFT) test developed by Witkin. The mathematical representation ability test is given through questions from the set material as much as three questions. In comparison, the interviews were conducted through interview guidelines.

The GEFT test consists of three groups of questions where the first group of questions consists of 7 items, and the second and third groups of questions each consist of 9 items. The first group of questions is given as an exercise for respondents to understand how the test works so it does not affect the score. Meanwhile, when students work on the second and third groups of questions, students will get a score. Students who answered correctly were given a score of 1; if they answered incorrectly, they were given a score of 0. For the first group of questions, they were given 5 minutes, then for the second and third groups of questions; they were each given 9 minutes.

This study's subjects were twenty Integrated Bilingual Junior High School students in class VIII-H who took the cognitive style test. After twenty students carried out the GEFT test, the participants were categorized into field-dependent, field-independent, and ambiverts (not dependent on FI and FD). Grouping calculations are based on the value of M ± standard deviation as a cut-off point (can be seen in Figure 1), where the value of M ± standard deviation is determined from the scores of twenty students (Setiawan et al., 2020).

![Figure 1. Cognitive style category (M: mean, SD: Standard Deviation)](image)

Figure 1 shows that if students score less than or equal to M – SD, they are in the field-dependent category. If students are at a value greater than or equal to M + SD, they are in the independent field category. Apart from these values, they are classified as ambivert or do not experience a tendency towards field-dependent and field-independent cognitive styles.
After the students completed the GEFT test, they were given questions to test their ability to represent the set topic mathematically. The indicators of mathematical representation ability can be seen in Table 1.

**Table 1. Mathematical Representation Indicator**

<table>
<thead>
<tr>
<th>Representation</th>
<th>Rated aspect</th>
<th>Student Responses</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual</td>
<td>Presenting answers through pictures, graphs or tables</td>
<td>Does not involve a visual representation to clarify the problem</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inappropriate or incorrect visual representation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The visual representation is appropriate or true</td>
<td>2</td>
</tr>
<tr>
<td>Symbol</td>
<td>Presenting answers using mathematical symbols or mathematical models</td>
<td>Does not state symbols or mathematical models</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stating a mathematical symbol or model incorrectly</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stating symbols or mathematical models correctly</td>
<td>2</td>
</tr>
<tr>
<td>Numeric</td>
<td>Present answers using numbers</td>
<td>Does not state the number to answer</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stating a number in the answer but wrong</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>State the number in the correct answer</td>
<td>2</td>
</tr>
<tr>
<td>Verbal</td>
<td>Present the answer in a word or sentence</td>
<td>There is no conclusion in the form of a sentence</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>There is a conclusion in the form of a sentence, but it is wrong</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>There is a conclusion in the form of a correct sentence</td>
<td>2</td>
</tr>
</tbody>
</table>

The grid questions to see students’ mathematical representation abilities are as follows:

**Table 2. Question Grid**

<table>
<thead>
<tr>
<th>Learning objective</th>
<th>Indicators of Competence Achievement</th>
<th>Question Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve contextual problems related to the characteristics of sets, types of sets, set operations, set relations, and venn diagrams.</td>
<td>Solve problems related to set operations</td>
<td>From the data of 25 students of class VIII-B, it is known that 20 students like drinking coffee and 13 students like milk. Determine a. how many students like coffee? b. how many students like milk? c. how many like both?</td>
</tr>
</tbody>
</table>
After conducting the representation ability test, researchers conducted interviews with subjects S1 and S2 who tend to have a field dependent cognitive style and S3 and S4 subjects who tend to have a field independent cognitive style. Interviews are used as data reinforcement to analyse the overall results of the test. The interview grids are as follows:

<table>
<thead>
<tr>
<th>Table 3. Mathematical representation skills interview grid</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Indicator</strong></td>
</tr>
<tr>
<td>Understand the problem</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Plan problem solving</td>
</tr>
<tr>
<td>Problem-solving</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Make a conclusion</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

To analyze the data, it was done by means of triangulation of data from field-dependent and field-independent cognitive style grouping test data. The results of mathematical representation ability tests and interview data were analyzed through Nvivo 12 Pro software. The results draw conclusions to answer the research objectives.

3. RESULTS AND DISCUSSION

3.1. Results

Based on the results of the cognitive style test through the GEFT test of 20 students in class VIII-H of the Integrated Bilingual Middle School with the groupings in Figure 1, the following data is obtained:

<table>
<thead>
<tr>
<th>Table 4. GEFT Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Category</strong></td>
</tr>
<tr>
<td>--------------------</td>
</tr>
<tr>
<td>Field dependent</td>
</tr>
<tr>
<td>Ambiverts</td>
</tr>
<tr>
<td>Field independent</td>
</tr>
</tbody>
</table>

Following are the results of the mathematical representation ability test and interviews with the four subjects.

1. **Students' mathematical representation ability with field dependent cognitive style S1.**

The following are the results of the answers to the mathematical representation ability test questions conducted by S1:
Figure 2. S1 representation ability test results

Figure 2 shows that S1 can answer questions with known writing stages, write what is asked, answer questions correctly, and provide concluding sentences. To solve S1 questions, use the ability of numerical mathematical representations and verbal representations. S1 shows numerical representation ability with the answers in Figure 2a, the numbers to calculate the number of students who like coffee and milk. S1 shows verbal representation ability in Figure 2b by writing words as a conclusion. The results of interviews conducted by researchers with S1 obtained information that S1 can explain the steps to solving set operations questions. S1 conveys a uniqueness in the sixth sentence. Namely, S1 uses a method that is easy to remember through a quick method.

P(1) : “how did you try to answer the question??”
S1(2) : “I answered by writing down what was known, then I answered what was asked”
P(3) : “whether the way to complete the answer according to what was taught by your teacher?”
S1(4) : “hmmm, I'm just writing down the quick method, ustadz, if you give an example in full”
P(5) : “why are you using the fast way?”
S1(6) : “yes, so that it’s simpler, Ustad and remember that I'm just a quick way”

From the data on the mathematical representation ability test results from Figure 2 and interviews at S1, information was obtained that S1 could solve questions with correct answers. The ability of mathematical representation shows results that lead to the tendency of numerical and verbal representation. So if viewed from the field-dependent cognitive style, the result is that the tendency for mathematical representation abilities is numerical and visual and prefers simple answers or fast ways.

2. The ability of students' mathematical representation with field dependent cognitive style S2.
The following are the results of the answers to the mathematical representation ability test questions conducted by S2:
Figure 3. S2 representation ability test results

Figure 3 shows that S2 has answered the representation ability test questions correctly. S1 writes down the answer by showing the numerical representation in Figure 3a and the verbal representation in Figure 3b. The ability of numerical representation can be seen from the numbers to calculate the answers of many students who like coffee and milk. The ability of verbal representation is shown in words in the concluding sentences written in the S2 answers. S2 did not write down the steps to answer math questions from what was known and what was asked. Word in conclusion. The results of interviews with S2 obtained information indicating that S2 could explain the steps for answering questions. However, S2 realized that he was answering questions by directly calculating the results asked. A uniqueness emerges from the interview results, shown in sentence number 8, S1 prefers the fast method for simpler reasons.

P(7) : “why do you prefer the fast way only?
S2(8) : “because it's easier to answer”
P(9) : “do you get used to answering questions like that?”
S2(10) : “not really ustad, sometimes I also use the long way if there really is no fast way”

Based on the results of data from Figure 3 and interviews from S2, the result was that S2 was able to answer the mathematical representation ability test questions correctly. These results also show the representation ability with numerical and verbal representation indicators. So if viewed from the field-dependent cognitive style, the results show that mathematical representation capabilities tend to be numerical and visual. Another characteristic is that the field-dependent prefers a quick way to find answers.

3. The ability of students' mathematical representation with field independent cognitive style S3.
The following are the results of the answers to the questions on the mathematical representation ability test conducted by the S3 subject:
Figure 4. The results of the S3 subject's representation ability test

In Figure 4, the results of the S3 representation ability test showed that the answers were correct, as well as demonstrating visual (4a), symbolic (4b), numerical (4c), and verbal (4d) representation abilities. Visual representation ability, S3 wrote Venn diagrams to answer questions. The subject's symbolic representation ability, S3 writes a mathematical model through algebra by making an example of the variable x. Numerical representation ability, the S3 uses numbers to find answers through number operations. In verbal representation ability, S3 subjects make conclusions from the results of the answers. The results of interviews with S3 have described the process of answering questions. Subject S3 explains in a coherent way how to answer questions. In one part of the interview, important information can be seen in sentence 12. The S3 followed the directions exemplified by his teacher when answering the questions. Like the following interview excerpt:

P(11): “did your teacher teach you that way?”
S3(12): “yes”
P(13): “at the end of the answer, do you write a conclusion?”
S3(14): “sure”

Based on the presentation of the results of the S3 subject, it was obtained data that the S3 subject was able to solve the questions correctly. The answers from the S3 subject showed the ability to represent mathematically using four indicators, namely visual, symbolic, numerical and verbal. If viewed from the cognitive style of subject S4 is a subject with a field independent cognitive style with a tendency to use all mathematical representations which include visual, symbolic, numerical and verbal. As well as the independent field cognitive style shows more following the directions exemplified by the teacher.

4. Students’ mathematical representation ability with field independent cognitive style subject S4
Following are the results of subject S4’s answers in answering the mathematical representation ability test.
Figure 5. The results of the S4 subject's representation ability test

Figure 5 shows the results of subject S4's answers. It can be seen that the answers are correct and show visual representation (5a), symbolic (5b), numerical (5c), and verbal (5d) representation abilities. The ability of visual representation can be seen in the answers with pictures of Venn diagrams. Symbolic abilities, S3 subjects write algebraic equations with the variable x. Numerical abilities are shown by the numbers in solving equations. The verbal representation ability is shown by writing conclusions with sentences, so the number of students who like both is eight. The interviews with subject S4 showed that subject S4 answered according to the mathematical procedure of knowing, being asked, answering, and so according to written answers. There is important information from the results of subject S4's answers that S4 answered according to what was exemplified by the teacher when answering the questions. This condition is shown in the interview excerpt in the 16th sentence.

P (15) : “did your teacher teach you how to solve it like you did?”
S4 (16) : “yes, I answered according to the example of my teacher”

Based on the exposure to the results of subject S4, data was obtained that subject S4 could solve the questions correctly. The answers from subject S4 show the ability to represent mathematically using all four indicators: visual, symbolic, numerical, and verbal. If viewed from the cognitive style of subject S4 is a subject with a field-independent cognitive style with a tendency to use all mathematical representations, which include visual, symbolic, numerical, and verbal, as well as the field-independent cognitive style showing characteristics that follow the conditions taught by the teacher.

3.2. Discussion

The results of the field-dependent and field-independent cognitive style tests, the results of the mathematical representation ability tests, and the results of the interviews can be shown in Table 4.
Table 5. Results of data analysis

<table>
<thead>
<tr>
<th>Cognitive Style</th>
<th>Mathematical Representation Ability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Visual</td>
</tr>
<tr>
<td>Field dependent</td>
<td></td>
</tr>
<tr>
<td>Not yet able to demonstrate visual representation ability</td>
<td>Not yet able to demonstrate the ability of symbol representation</td>
</tr>
<tr>
<td>Field independent</td>
<td>Can demonstrate visual representation skills</td>
</tr>
</tbody>
</table>

Table 4 shows that in subjects with field-dependent cognitive styles in answering test questions, representational abilities only appear in numerical and verbal representations but do not generate visual representations and symbols. While subjects with field-dependent cognitive style in answering test questions, mathematical representation abilities show all four indicators, which include visual, symbolic, numerical, and verbal.

Students' mathematical representation abilities in terms of cognitive styles show significant differences, as seen from the findings in this study. Differences in indicators of mathematical representation abilities between field-dependent cognitive styles and field-independent cognitive styles occur due to differences in students' thinking styles (Septian et al., 2020; Widakdo, 2017). Students whose cognitive style is field dependent tend to answer practically so that results are obtained, only sometimes following the method given by the teacher when solving math problems. Meanwhile, the independent field cognitive style in solving mathematical problems imitates what is exemplified by the teacher so that it includes all indicators of representational abilities.

The results of the study show that the ability to solve problems in the independent field cognitive style is more diverse in using mathematical representations, while the field-dependent cognitive style only uses several mathematical representations in solving mathematical problems (Ratuanik, 2018; Rofiq et al., 2021; Yusrina & Masriyah, 2018). In addition, the results of other studies also state that students' cognitive styles are very important to pay attention to in the learning process so that students can solve math problems correctly (Son et al., 2020). So that students' representation abilities can be seen from their modeling abilities as indicated by visual representations and symbols and being able to make interpretations through the form of numerical and verbal representations (NCTM, 2000).

With differences in students' mathematical representation abilities, teachers are expected to be able to prepare learning designs with various kinds of mathematical representation abilities (Prihandhika et al., 2022). Moreover, the teacher also pays attention to the differences in the cognitive style of each student. Students' cognitive style is very important to note in the learning process so that students can solve mathematical problems. This situation is in line with the implementation of the independent curriculum in Indonesia, where in the learning process, the learning approach through differentiated instruction or differentiated learning, teachers can group students based on the teacher's initial abilities and can also group based on field-
dependent and field-independent cognitive styles. So that all students can achieve learning objectives by reviewing the differences in students (Abu Bakar & Ali, 2018; Faradiba, 2019, 2022; Faradiba et al., 2023; Hayes & Allinson, 1994).

4. CONCLUSION

Through data analysis from the results of students' cognitive style tests, tests of students' representation abilities, and interviews conducted at the Integrated Bilingual Middle School Class VIII for the 2022/2023 academic year, it was concluded that mathematical representation abilities with field-dependent cognitive style are good enough to represent questions about the set, because only able to use some indicators of mathematical representation, namely numerical and verbal. Meanwhile, mathematical representation with a field-independent cognitive style is very good at representing the questions about the set because it uses all indicators of mathematical representation, namely visual, symbolic, numeric, and verbal. The development of students' mathematical representation abilities so that they are more developed cannot be separated from the teacher's role as a facilitator to use various mathematical representations when solving mathematical problems to achieve learning goals. Suggestions from this study It is expected that field-dependent and field-independent cognitive styles are used as one factor to see differences in student profiles so that teachers can plan to learn according to the differences of each student so that all students can achieve learning objectives.

REFERENCES


