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THE ABSTRACTION PROCESS OF NINTH-GRADE STUDENTS IN SOLVING POLYHEDRAL GEOMETRY PROBLEMS BASED ON LEARNING STYLES

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ABSTRACT

This study aims to describe the abstraction process of ninth-grade students in solving problems involving flat-sided three-dimensional shapes, as viewed from the perspective of learning styles based on the RBC+C model. This study employs a qualitative approach, in the form of a description of the abstraction process of ninth-grade students at SMP Negeri 14 Palu. The research subjects consisted of three ninth-grade students at SMP Negeri 14 Palu who were selected using purposive sampling based on the results of a learning style questionnaire (visual, auditory, and kinesthetic). Data collection began with the distribution of the learning style questionnaire, followed by a written test on flat-sided three-dimensional shapes to measure RBC+C epistemic actions, as well as semi-structured interviews to explore the students' responses in greater depth. The research findings indicate that all three subjects were able to complete the cognitive activity stages of the RBC+C model, though with varying characteristics and levels of success. Two subjects successfully reached a stable Consolidation stage, while the other subject demonstrated only very limited progress. The dominant thinking barriers occurred during the Building-with stage, taking the form of algebraic misconceptions when reversing formulas and manipulating the notation of mathematical operations. These findings provide insights for educators in understanding the diversity of students' learning styles and anticipating their cognitive obstacles when designing geometry instruction.

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1. INTRODUCTION

Mathematics, as one of the core subjects in junior high school, plays an essential role in developing students' logical, critical, and abstract thinking skills. One of the topics in the ninth-grade mathematics curriculum that requires abstract thinking is geometry, particularly plane and solid geometry. This topic requires students not only to understand geometric shapes visually but also to mentally visualize and manipulate their representations as part of the abstraction process. However, in practice, many students still experience difficulties in understanding problems presented in the form of diagrams or those requiring spatial imagination to obtain solutions (Fitrianti, 2025).

Hershkowitz et al. (2001) define abstraction as an activity of reorganizing previously acquired mathematical knowledge through vertical mathematization to construct new mathematical knowledge. Abstraction is an essential cognitive process through which mathematical concepts are formed (Breive, 2022). It is also a process of constructing mental structures by establishing relationships among mathematical objects and transforming these relationships into representations that are independent of the original objects. In this process, previously acquired mathematical concepts are reorganized to reconstruct new mathematical structures (Dewi, 2023).

Despite its importance, mathematics continues to be perceived by many students as one of the most challenging school subjects. Many students also perceive mathematics as an intimidating subject. This perception is influenced by two main factors: the abstract nature of mathematics itself and the way it is taught (Mubarik et al., 2026). One reason for this negative perception is that mathematics is inherently abstract. Students often experience difficulties in understanding and solving geometry problems because their abstraction processes have not yet developed optimally (Nisaâ, 2018). Various factors may influence this process, one of which is students' learning styles. Visual, auditory, and kinesthetic learning styles influence how students acquire and process mathematical information, as well as how they construct mental representations of geometric solids (Putri et al., 2022).

A study conducted by Juliyansah & Hakim, (2023) found that students experienced difficulties in visualizing solutions while working on geometry tests. They often needed to read the questions repeatedly before they could understand and mentally visualize the intended problem. Furthermore, Budiarto et al. (2024) reported that the epistemic actions in the Recognizing–Building-with–Constructing (RBC) framework do not necessarily occur in a fixed sequence. Instead, students may move flexibly between recognizing, building-with, constructing, and consolidating throughout the problem-solving process. In addition, Hapsari & Masduki, (2024) found that students with visual and kinesthetic learning styles demonstrated all computational thinking indicators, including abstraction, pattern recognition, algorithmic thinking, and generalization. In contrast, students with auditory learning styles demonstrated only three indicators: abstraction, pattern recognition, and algorithmic thinking.

Although previous studies have investigated difficulties in geometry learning, the RBC framework, and learning styles separately, no study has comprehensively integrated these three aspects. This research gap forms the basis of the present study. Therefore, this study aims to integrate the abstraction process, learning styles, and polyhedral geometry among junior high school students to provide a deeper understanding of students' thinking processes. The findings are also expected to contribute to the development of more effective mathematics teaching strategies for classroom practice.

2. METHOD

The data were collected from one ninth-grade class consisting of 29 students through the administration of an adapted O'Brien learning style questionnaire, a validated written task, and in-depth interviews. This study employed a qualitative approach to describe the abstraction process of ninth-grade students at SMP Negeri 14 Palu in solving polyhedral geometry problems based on their learning styles. The study was conducted at SMP Negeri 14 Palu during the second semester of the 2025/2026 academic year. The research subjects consisted of three students selected through purposive sampling based on the results of the learning style questionnaire, representing the visual, auditory, and kinesthetic learning style categories. One student was selected to represent each learning style.

The primary research instrument was the researcher. The supporting instruments included a learning style questionnaire adapted from O'Brien (1989), a written task on polyhedral geometry (cuboids and cubes), and a semi-structured interview guide. The learning style questionnaire was administered to identify students with visual, auditory, and kinesthetic learning styles. The written task was designed to examine students' abstraction processes, and the instrument was validated by mathematics education experts to ensure its appropriateness and alignment with the research objectives. Based on the validators' recommendations, the researcher revised the instrument by selecting one problem that fulfilled the RBC+C indicators and adding a narrative context to make the problem easier for students to understand. The interviews were conducted to obtain a deeper understanding of students' abstraction processes while solving the given problem. The interview guide was developed based on the RBC+C epistemic action indicators, namely Recognizing, Building-with, Construction, and Consolidation, to ensure that the collected data systematically reflected students' abstraction processes.

The data collection process was carried out in several interconnected stages. First, the learning style questionnaire was administered to all students in one ninth-grade class to identify their dominant learning styles. Based on the questionnaire results, three students representing the visual, auditory, and kinesthetic learning styles were selected as the research subjects. Next, the selected students completed a written task on polyhedral geometry to examine their abstraction processes based on the RBC+C epistemic action indicators. Finally, semi-structured interviews were conducted to explore and clarify the students' responses to the written task.

The trustworthiness of the data was established through the credibility criterion by ensuring that the findings accurately represented the participants' perspectives. One of the techniques employed to establish credibility was member checking. The data were analyzed through data condensation of the written task and interview results based on the epistemic action indicators. The findings were then presented in the form of narrative descriptions and interview excerpts, followed by drawing conclusions to ensure that the interpretations accurately represented students' abstraction processes in solving polyhedral geometry problems. Table 1 presents the epistemic action indicators adapted from the RBC+C model (Recognizing, Building-with, Construction, and Consolidation), which served as the analytical framework for examining students' abstraction processes in this study.

Table 1. Epistemic Action Indicators

Epistemic Action	Operational Definition
Recognizing (R)	Recalling previously acquired mathematical constructs (properties and definitions of each quadrilateral) that are relevant to the given task, particularly the relationships among quadrilaterals.
Building-with (B)	Combining existing mathematical elements (properties and definitions of each quadrilateral) to develop a new concept regarding the relationships among quadrilaterals.
Construction (C)	Reorganizing and integrating previously acquired mathematical constructs to construct a new mathematical structure concerning the relationships among quadrilaterals.
Consolidation (+C)	Reinforcing the newly constructed knowledge to facilitate its application in subsequent mathematical activities and problem-solving situations.

Source: Budiarto et al. (2024)

3. RESULTS AND DISCUSSION

3.1. Results

This study aims to describe the abstraction process of ninth-grade students at SMP Negeri 14 Palu in solving polyhedral geometry problems based on their learning styles using the RBC+C model. The abstraction processes of the three research participants were analyzed according to the stages of the RBC+C epistemic actions, namely Recognizing, Building-with, Construction, and Consolidation, and are presented as follows.

Participant GV1

Based on the analysis of the written task and the interview with participant GV1, an overview of the participant's abstraction process in solving polyhedral geometry problems was obtained. At the Recognizing stage, GV1 demonstrated the ability to identify the facts and basic concepts relevant to the given problem.

PGV103 : Could you explain what you understood after reading this problem?
GV104 : After reading the problem, I understood that a teacher made a cuboid-shaped box with a volume of $288\ cm^3$, a length of $12\ cm$, and a width of $8\ cm$. **(R)**

Based on excerpt GV104, participant GV1 was able to identify and state the information provided in the problem, namely that the problem involved a cuboid with a volume of $288\ cm^3$, a length of $12\ cm$, and a width of $8\ cm$. Figure 1 shows that the participant understood the meaning of the problem and successfully recognized the given information required to solve it.

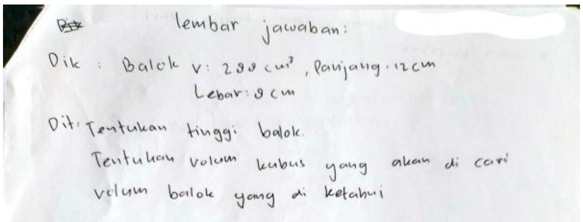

Figure 1. Recognizing Stage of Participant GV1

PGV111 : Have you ever encountered a similar problem before?
GV112 : Yes. I have encountered a similar problem while studying at home using a digital learning application. **(R)**

Furthermore, excerpt GV112 indicates that participant GV1 had previously encountered a similar problem during independent learning at home using a digital learning application. This suggests that the participant was able to relate the characteristics of the current problem to prior learning experiences. Based on these findings, GV1 fulfilled the Recognizing indicator by identifying the information provided in the problem, recognizing the required data, recalling the relevant mathematical concepts, and connecting the problem with previous digital learning experiences.

At the Building-with stage, participant GV1 attempted to apply mathematical formulas and computational procedures to solve the problem, although errors were still evident in the calculation process.

PGV113 : Could you explain how you determined the height of the cuboid?
GV114 : First, I found the height of the cuboid. The volume is $288\ cm^3$ and the formula for the volume of a cuboid is $p \times l \times t$. To find the height, I used the formula $t = p \times l \div volume$. So, I multiplied the length $(12\ cm)$ by the width ($8\ cm$), then divided the result by the volume $96 \div 288 = 3\ cm$. **(B)**
PGV117 : How did you obtain this value?
GV118 : After getting the result, I divided 96 by 288 and obtained 3, or $9\sqrt{288 = 3\ cm.}$ **(B)**

Based on excerpts GV114 and GV118, participant GV1 demonstrated the Building-with (B) stage by actively applying the information provided in the problem to perform the required calculations. However, the cognitive analysis revealed an error in the algebraic manipulation of the formula. The participant transformed the cuboid volume formula, length × width × height, into $t = p \times l \div volume$, which is mathematically incorrect because the numerator and denominator were reversed. The correct formula should be $t = volume \div p \times l$. As a result, the participant calculated 96 ÷ 288 but then concluded that the height was 3 cm through an inconsistent arithmetic procedure, including the appearance of an expression resembling a square-root operation in excerpt GV118.

The participant also appeared to recognize that the division did not produce a whole number and therefore attempted to adjust the procedure by writing the calculation in a long-division format using a symbol resembling a radical sign $9\sqrt{288,}$ ultimately concluding that the height was 3 cm. In doing so, the participant reversed the logic of the division to match the expected height value, whereas the correct calculation should have been $288 \div 96 = 3$. Although the algebraic structure and computational procedure were formally incorrect, as illustrated in Figure 2, this activity was still categorized as Building-with from the perspective of the RBC+C framework. This classification is appropriate because the participant actively applied previously learned mathematical procedures to support the solution process and continued using operational mathematical knowledge to determine the height of the cuboid.

PGV119: After determining the height of the cuboid, what did you do next?
GV120: After finding the height of the cuboid, I calculated the volume of the cube. The formula for the volume of a cube is $s \times s \times s$. The side length of the cube is the same as the height of the cuboid, which is 3 cm. Therefore, $3 \times 3 \times 3 = 27\ cm^3$. **(R, B)**

Excerpt GV120 shows a transition in the participant's thinking process. The Recognizing (R) action occurred when the participant recalled the formula for the volume of a cube, namely $s \times s \times s$. This was followed by the Building-with (B) action, in which the participant used the previously determined height of the cuboid (3 cm) as the side length of the cube and applied repeated multiplication ($3 \times 3 \times 3$) to obtain the cube's volume of $27\ cm^3$.

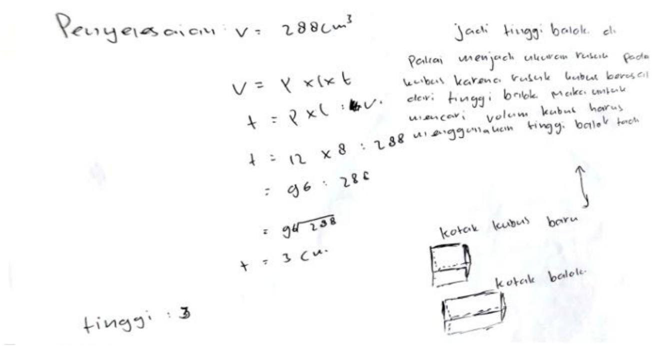

Figure 2. Building-with Stage of Participant GV1

At the Construction stage, participant GV1 demonstrated a more advanced level of understanding throughout the problem-solving process. The participant not only applied mathematical formulas during the Building-with stage but also integrated the available information to construct new conceptual understanding.

PGV133: Did you gain any new understanding after solving this problem?
GV134: Yes. I learned how to determine the height of a cuboid when only the volume, length, and width are given. **(C)**

Excerpt GV134 demonstrates the Construction (C) epistemic action. The participant experienced a restructuring of knowledge by consciously developing a new understanding after completing the task. This new understanding involved a strategy for determining the height of a cuboid when one of its dimensions is unknown. The participant's awareness of having developed this new concept indicates that the thinking process had progressed beyond procedural application toward the construction of a new cognitive structure. As illustrated in Figure 3, participant GV1 successfully internalized this newly acquired knowledge into a stable cognitive structure that can be applied in future problem-solving situations.

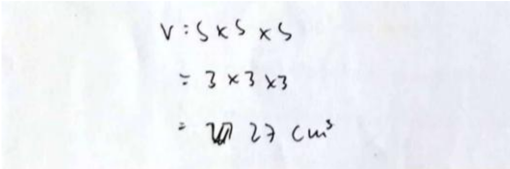

Figure 3. Construction Stage of Participant GV1

At the Consolidation stage, participant GV1 demonstrated that the newly acquired knowledge had been successfully internalized into a stable cognitive structure that was ready to be applied in future problem-solving situations. The participant's Construction and Consolidation abilities were evidenced by the following interview excerpts.

PGV139 : Are you confident that your answer is correct?
GV140 : Yes, I am. **(+C)**
PGV147 : What did you understand after completing this problem?
GV148 : I understood that a cuboid and a cube have several similarities. **(C)**
PGV149 : If you were given a similar problem, do you think you could solve it again?
GV150 : Yes, I could. **(+C)**

Based on excerpt GV148, participant GV1 fulfilled the Construction (C) indicator by developing a generalized conceptual understanding after completing the entire problem-solving process. Rather than viewing the cuboid and the cube as two separate geometric solids, the participant constructed a new understanding of the structural relationships and shared characteristics between the two shapes. This understanding emerged through the integration of ideas developed throughout the problem-solving process.

Furthermore, as illustrated in Figure 4, the participant demonstrated a stable level of understanding, as reflected in the Consolidation (+C) indicators shown in excerpts GV140 and GV150. The participant's confident response, “Yes, I am,” regarding the correctness of the solution, together with the immediate response, “Yes, I could,” when asked about solving a similar problem in the future, indicates that the newly constructed knowledge had become well established. The participant had consolidated the understanding of the relationship between the dimensions of a cuboid and a cube into long-term memory, resulting in greater confidence and cognitive readiness to apply this knowledge when solving similar geometry problems in the future.

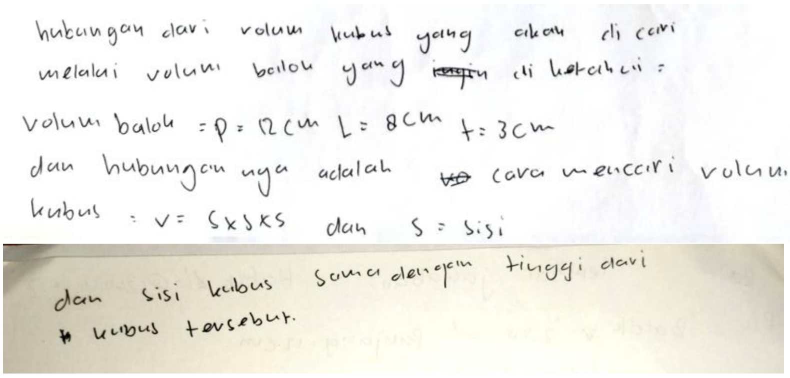

Figure 4. Consolidation Stage of Participant GV1

Participant GA1

Based on the analysis of the written task and the interview with participant GA1, a clear description of the participant's thinking process in solving polyhedral geometry problems was obtained. At the Recognizing stage, participant GA1 demonstrated the ability to identify relevant facts and fundamental concepts. This was evidenced by the following interview excerpts.

PGA103: Could you explain what you understood after reading this problem?
GA104: After reading the problem, I understood that I had to make a new box in the shape of a cube. The problem stated that the side length of the cube had to be the same as the height of the cuboid. **(R)**

Based on excerpt GA104, participant GA1 was able to identify and describe the information provided in the problem, namely that a new cube had to be constructed with a side length equal to the height of the cuboid. This indicates that the participant understood the meaning of the problem and recognized the relationship between the height of the cuboid and the side length of the cube required to solve it.

PGA111: Have you ever encountered a similar problem before?
GA112: Yes. I have seen many similar problems on TikTok. **(R)**

Based on excerpt GA112, participant GA1 stated that similar problems had previously been encountered on the digital platform TikTok. As illustrated in Figure 5, the participant was able to relate the given problem to previous learning experiences. Therefore, participant GA1 fulfilled the Recognizing indicator by identifying the information presented in the problem, recalling relevant mathematical concepts, and connecting the problem with prior learning experiences.

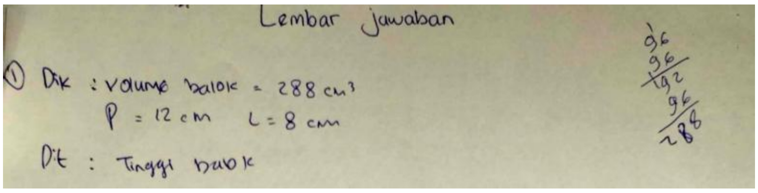

Figure 5. Recognizing Stage of Participant GA1

At the Building-with stage, participant GA1 began using the previously identified information, concepts, and formulas to solve the given problem. The participant applied the relevant formula and carried out the calculations step by step to determine the height of the cuboid before continuing the solution to obtain the information required to calculate the volume of the cube. This process is illustrated in the following interview excerpt.

PGA125: Could you explain the steps you used to solve the problem until you obtained your answer?
GA126: The known information is that the volume of the cuboid is 288 cm^3, the length is 12 cm, and the width is 8 cm. The unknown is the height of the cuboid. I used the volume formula, volume $= p \times l \times t$, then $t = p \times l \div t$. So, height $= 12\ cm \times 8\ cm \div 288\ cm^3 = 96 \div 288 = 3\ cm$. **(B)**

In the written task, participant GA1 initially wrote the incorrect division operation $96 \div 288$, indicating an error in the calculation procedure. The participant then added a visual modification by writing $96\sqrt{288}$, which resembled a radical expression. After this mathematically inconsistent sequence of symbols, the participant directly wrote the final answer, $9\ cm.$

Based on the RBC+C framework, these findings indicate that participant GA1 was able to apply an appropriate solution procedure to determine the height of the cuboid. However, as shown in Figure 6, the inconsistency in the written calculations suggests that the participant focused more on following procedural steps than on explaining the conceptual relationships underlying the calculations. Consequently, the participant's performance at this stage primarily reflected procedural application rather than conceptual understanding. The participant appeared to manipulate mathematical symbols mechanically by reversing the division operation and inserting a radical-like symbol to make the solution appear mathematically formal. The main objective was to obtain the final answer of 3, which the participant had already anticipated, rather than to develop a new conceptual understanding. This finding indicates that the participant relied on external procedural patterns without constructing new mathematical knowledge.

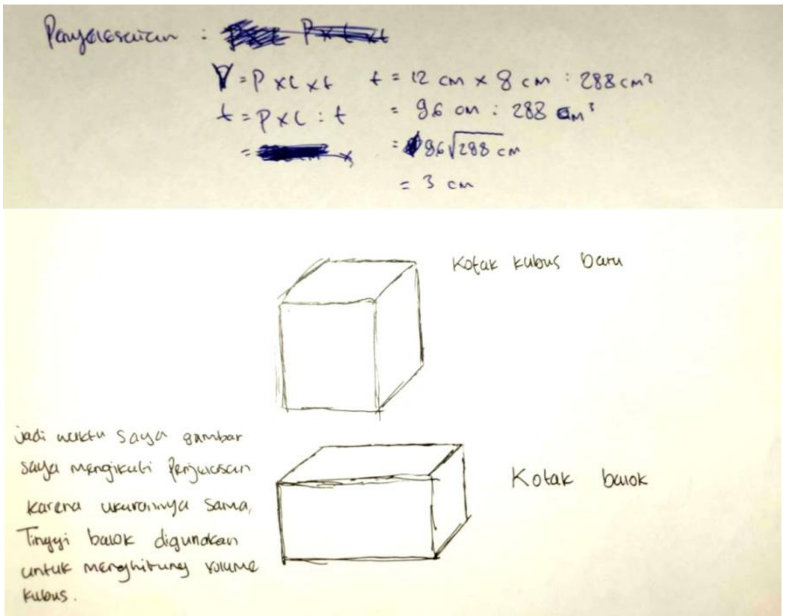

Figure 6. Building-with Stage of Participant GA1

At the Construction stage, participant GA1 did not demonstrate the expected indicator. Instead, the participant relied on previously learned concepts and procedures to solve the problem without showing evidence of constructing or developing new conceptual understanding. This is illustrated in the following interview excerpt.

PGA129: Based on the drawing you made, could you explain the relationship between the height of the cuboid and the side length of the cube?
GA130: When I drew it, I followed the explanation because the measurements were the same. The height of the cuboid is used to calculate the volume of the cube. **(B)**

Excerpt GA130 indicates that participant GA1 explained that the drawing was based on the information provided in the problem because the dimensions were the same.

The participant acknowledged that drawing the figure and relating the dimensions were based on explanations learned previously. This suggests that the relationship between the height of the cuboid and the side length of the cube was not independently developed but was derived from prior knowledge. As illustrated in Figure 7, participant GA1 tended to reuse previously learned procedures without demonstrating the construction of a new conceptual relationship. Therefore, the participant's responses did not meet the characteristics of the Construction stage, which emphasizes the independent development or reconstruction of conceptual understanding.

The absence of the Construction stage was further supported by the participant's acknowledgment of having previously encountered and learned a similar solution strategy, as shown in the following interview excerpt.

PGA133: Did you gain any new understanding after solving this problem?
GA134: No. **(C)**

Based on excerpt GA134, the participant did not demonstrate the characteristics of the Construction stage, which requires the independent development of new understanding. The participant was only able to determine the volume of the cube by using the previously obtained height of the cuboid. Although this value was applied to calculate the cube's side length and volume, the process did not result in the construction of new conceptual understanding. Consequently, the Construction indicator was not achieved.

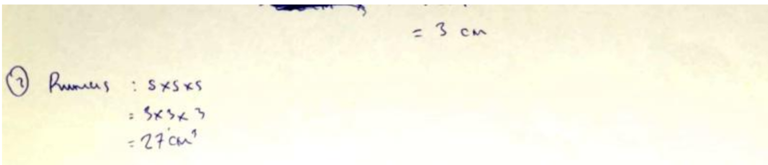

Figure 7. Construction Stage of Participant GA1

The Consolidation stage refers to the phase in which the knowledge constructed by the participant becomes more stable and firmly established. However, because participant GA1 did not demonstrate the Construction (C) stage in the previous phase, the analysis at this stage focused on the extent to which the participant had consolidated the procedural knowledge used to solve the problem.

PGA149: If you were given a similar problem, do you think you could solve it again?
GA150: Yes, I could. **(+C)**

Based on excerpt GA150, participant GA1 expressed confidence in the ability to solve a similar problem if presented with one in the future. From the perspective of the RBC+C framework, the interview findings indicate that the participant successfully consolidated the procedural skills used during the problem-solving process. As illustrated in Figure 8, the participant was able to consistently recall, explain, and apply the solution procedures. The participant also demonstrated confidence in solving similar problems because these procedures had been retained in memory.

Nevertheless, the consolidation observed in participant GA1 was limited to the Building-with (B) stage rather than the Construction (C) stage. This is because the participant did not demonstrate the construction of new conceptual understanding during the previous stage. Consequently, although GA1 successfully consolidated the problem-solving procedures, the Consolidation (+C) stage, which reflects the reinforcement of newly constructed knowledge or concepts, was not achieved.

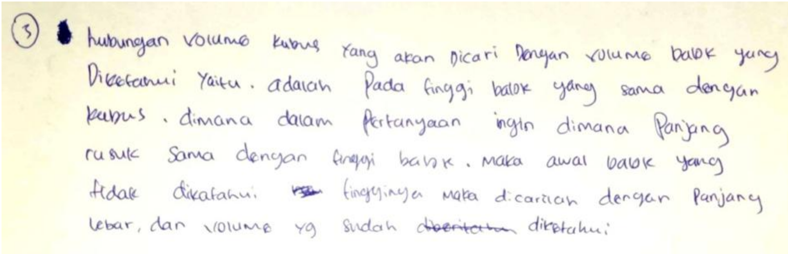

Figure 8. Consolidation Stage of Participant GA1

Participant GK1

Based on the analysis of the written task and the interview with participant GK1, an overview of the participant's thinking process in solving polyhedral geometry problems was obtained. At the Recognizing stage, participant GK1 demonstrated the ability to identify the facts and fundamental concepts relevant to the given problem. This ability is illustrated in the following interview excerpts.

PGK103: Could you explain what you understood after reading this problem?
GK104: What I understood after reading the problem was that the volume is 288 cm^3, the length is 12 cm, and the width is 8 cm. The teacher plans to make a new box in the shape of a cube, where the side length of the cube is the same as the height of the cuboid. **(R)**
PGK109: What formula or concept did you recall while solving the problem?
GK110: I remembered the formula for finding the height of the cuboid, so I immediately multiplied $12 \times 8 = 96$. Then I divided the cuboid's volume 288 cm^3 by 96 to get 3 cm. After that, I remembered that the formula for the volume of a cube is $s \times s \times s$, so I calculated $3 \times 3 \times 3 = 27\ cm^3$ **(R)**

Based on excerpts GK104 and GK110, participant GK1 successfully identified the information provided in the problem and recalled the mathematical knowledge required to solve it, including the formulas for determining the height of a cuboid and the volume of a cube (side × side × side). As illustrated in Figure 9, the ability to recall these relevant concepts enabled the participant to connect the information presented in the problem with the mathematical procedures required to solve it.

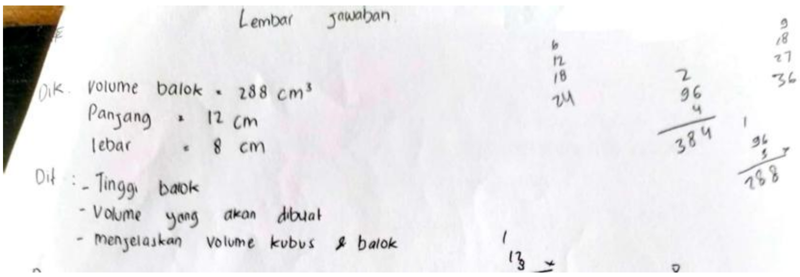

Figure 9. Recognizing Stage of Participant GK1

At the Building-with stage, participant GK1 began using the information, concepts, and formulas that had been previously recognized to solve the given problem.

The participant then continued the solution process to obtain the information required to calculate the volume of the cube. This is illustrated in the following interview excerpts.

PGK121: How did you determine the volume of the cube?
GK122: The problem already gave the volume of the cuboid, 288 cm^3, so I divided it by the height of the cuboid, which was 96 cm **(B)**
PGK123: Why can the height of the cuboid be used as the side length of the cube?
GK124: Because it is stated in the problem that the side length of the cube is the same as the height of the cuboid. **(B)**

Based on excerpt GK122, the participant made an error when explaining the solution verbally by referring to 96 cm as the height of the cuboid. In fact, 96 was the product of the cuboid's length and width, representing the area of its base. Nevertheless, excerpts GK122 and GK124 indicate that the participant was applying the solution procedure by connecting the given information. The participant correctly understood the key condition in the problem, namely that the height of the cuboid would be used as the side length of the cube to calculate its volume.

PGK125: Could you explain the steps you used to solve the problem until you obtained your answer?
GK126: First, I wrote down all the information given in the problem, including the volume, length, and width of the cuboid. Then I calculated the height by multiplying the length and width, $12 \times 8 = 96$. After that, I divided the cuboid's volume by 96 to obtain the height, which was 3 cm. Finally, I substituted this value into the cube volume formula, $s \times s \times s$ $3 \times 3 \times 3 = 27$ cm^3 **(R, B)**

Based on excerpt GK126, participant GK1 demonstrated the ability to apply the solution procedures in a systematic manner. The participant first multiplied the length and width of the cuboid $12 \times 8 = 96$, then divided the cuboid's volume (288 cm^3) by this result to obtain a height of 3 cm. Next, the participant used this value as the side length of the cube and substituted it into the cube volume formula, $side \times side \times side$, to obtain the final answer of 27 cm^3.

It should be noted that, in excerpt GK126, the participant's spontaneous recall of the cube volume formula ($s \times s \times s$) represents the Recognizing (R) stage. As illustrated in Figure 10, the participant recalled previously learned mathematical knowledge rather than constructing a new formula. Overall, the activities shown in excerpt GK126 indicate that participant GK1 was able to apply appropriate and systematic procedures to solve the given problem.

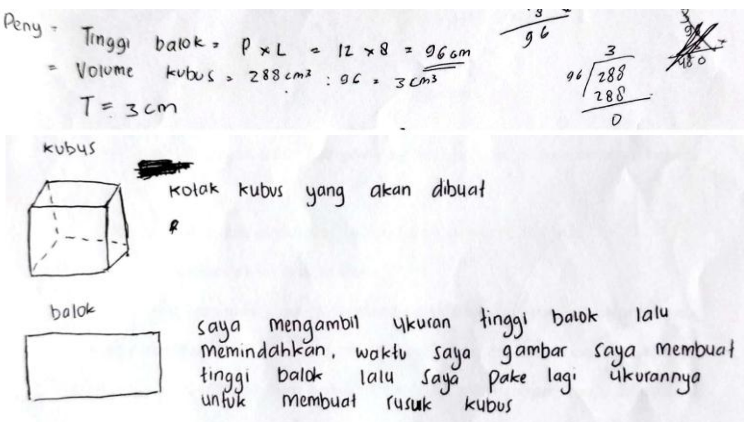

Figure 10. Building-with Stage of Participant GK1

At the Construction stage, participant GK1 successfully demonstrated the characteristics of Construction. The participant not only applied previously learned concepts but also developed a conceptual understanding of the logical relationship between the geometric elements presented in the problem. This is illustrated in the following interview excerpts.

PGK133: Did you gain any new understanding after solving this problem?
GK134: Yes. I learned that the side length of a cube can be determined from the height of the cuboid because the two measurements are related. **(C)**
PGK135: If the dimensions of the cuboid changed, would the volume of the cube also change?
GK136: Yes. **(+C)**

Based on excerpt GK134, participant GK1 clearly demonstrated the development of a new conceptual understanding after completing the problem. The participant recognized that a dimension of the cuboid, namely its height, could be used to determine the side length of the cube because of the relationship established in the problem. This finding indicates that the participant had moved beyond simply applying formulas and had constructed a new understanding of the relationship between the two geometric solids.

Interestingly, the response in excerpt GK136 can be categorized as part of the Consolidation stage. The participant demonstrated that the newly constructed knowledge had become more stable by recognizing that the relationship between the cuboid and the cube would remain valid even if the dimensions of the cuboid changed. As illustrated in Figure 11, participant GK1 logically explained that changes in the dimensions of the cuboid would lead to corresponding changes in the volume of the cube, demonstrating a strengthened understanding of the relationship between the two geometric solids.

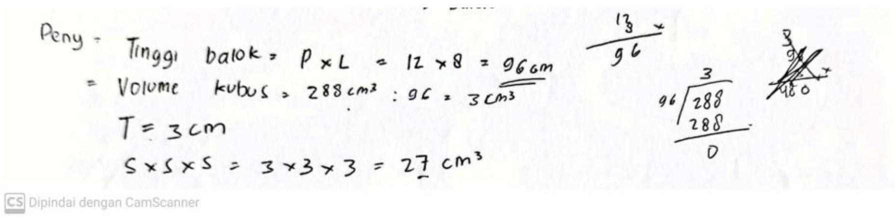

Figure 11. Construction Stage of Participant GK1

At the Consolidation stage, participant GK1 demonstrated strong evidence of Consolidation. This is illustrated in the following interview excerpt.

PGK141: How did you check your answer after completing the problem?
GK142: I checked my answer again and reviewed the parts that I was not completely sure about. **(+C)**

Excerpt GK142 shows that participant GK1 was able to explain the strategy used to verify the solution. The participant reviewed each step of the solution and recalculated the parts that were considered uncertain. This indicates an effort to monitor and evaluate the problem-solving process in order to ensure the accuracy of the final answer.

PGK147: What did you understand after completing this problem?
GK148: I have a better understanding of the relationship between the height of the cuboid and the side length of the cube. **(+C)**

PGK149: If you were given a similar problem, do you think you could solve it again?
GK150: Yes, I could. **(+C)**

Excerpt GK148 indicates that the participant developed a deeper understanding after completing the problem, particularly regarding the relationship between the height of the cuboid and the side length of the cube. Furthermore, excerpt GK150 demonstrates that participant GK1 successfully reached the Consolidation stage. As illustrated in Figure 12, the participant's confidence in solving a similar problem indicates that the newly acquired knowledge was not only applied during the current task but had also been retained and was ready to be applied in future problem-solving situations.

Overall, participant GK1 fulfilled the Consolidation indicator by reflecting on the problem-solving process, independently evaluating the solution, strengthening the understanding of the relationships among mathematical concepts, and demonstrating readiness to apply this knowledge when solving similar mathematical problems.

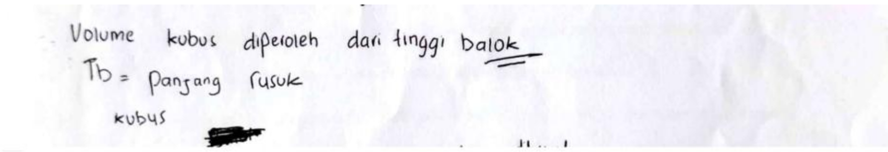

Figure 12. Consolidation Stage of Participant GK1

3.2. Discussion

The findings of this study indicate that ninth-grade students engaged in various epistemic actions while solving polyhedral geometry problems, as classified in the RBC+C model, including Recognizing, Building-with, Construction, and Consolidation. However, the sequence and characteristics of these stages differed among the three participants representing visual, auditory, and kinesthetic learning styles. These differences reflect varying levels of success in constructing new conceptual understanding, overcoming computational difficulties, and applying problem-solving strategies. The findings suggest that geometry instruction should be adapted to students' learning styles in order to better support their abstraction processes when solving mathematical problems.

3.2.1 Error Analysis of Participant GV1

Based on the analysis of the written task and interview data, participant GV1 demonstrated well-developed performance at the Recognizing stage of the RBC+C model. The participant's cognitive awareness was evident in the ability to systematically identify the given information, including the volume, length, and width of the cuboid, as well as to accurately recall the formulas for the volume of a cuboid $v = p \times l \times t$ and the volume of a cube ($v = s \times s \times s$). As a participant with a visual learning style, GV1 also demonstrated the ability to relate the given problem to previous independent learning experiences using digital learning resources at home. This strong ability to recall prior knowledge provided an important foundation for progressing to the subsequent stages of abstraction. These findings are consistent with Hershkowitz et al., (2001), who define Recognizing as the process through which students identify previously acquired knowledge that is relevant to solving the mathematical task at hand.

At the Building-with stage, participant GV1 actively applied the information provided in the problem to determine the height of the cuboid, although errors occurred in the algebraic manipulation. The participant incorrectly rearranged the volume formula as $t = p \times l \div volume$ and wrote a long-division operation resembling a radical expression $(9\sqrt{288})$ in an attempt to obtain the value of 3 cm. Although these procedural and arithmetic errors were evident, the activity was still classified as Building-with because the participant applied the mathematical rules that had been previously learned to maintain the problem-solving process. After obtaining the height of the cuboid (3 cm), the participant demonstrated relational reasoning by using this value as the side length of the cube and systematically calculating its volume $(3 \times 3 \times 3 = 27\ cm^3)$. This finding suggests that difficulties in algebraic manipulation did not hinder the participant's ability to understand the conceptual relationship between the two geometric solids. This finding is consistent with Budiarto et al. (2024), who reported that the epistemic actions in the RBC framework do not occur in a rigid linear sequence. Instead, students may repeatedly return to the Recognizing stage while engaging in Building-with to recall fundamental concepts before constructing more complex geometric relationships.

At the Construction stage, participant GV1's thinking process developed beyond the application of formulas toward the construction of new conceptual understanding. Through the drawing produced during the task, the participant developed a strategy for determining the height of the cuboid using the information provided in the problem. The participant also concluded that the cuboid and the cube were conceptually related, particularly in terms of their dimensions and structural elements.

At the Consolidation stage, participant GV1 demonstrated that this conceptual understanding had become more firmly established. When asked about the effect of changing the dimensions of the cuboid, the participant confidently explained that such changes would also affect the volume of the cube because the two geometric solids were related. This confidence was consistently reflected throughout the interview, during which the participant expressed confidence in the correctness of the solution and believed that a similar problem could be solved successfully in the future. These findings indicate that the participant had successfully consolidated the conceptual relationship between the cuboid and the cube, enabling this knowledge to be applied in solving similar mathematical problems.

3.2.2 Error Analysis of Participant GA1

At the Recognizing stage, participant GA1 demonstrated a good understanding of the given problem. The participant was able to identify the main objective of the task, namely constructing a cube whose side length was determined by the height of the cuboid, which first had to be calculated. Before beginning the calculations, the participant carefully considered the steps required to solve the problem. Participant GA1 also successfully recalled the formulas needed to determine the height of the cuboid and calculate the volume of the cube. As a participant with an auditory learning style, GA1 related the problem to explanations previously heard through the social media platform TikTok. This prior learning experience helped the participant recognize the characteristics of the problem and recall the mathematical knowledge required to solve it.

At the Building-with stage, participant GA1 actively applied the relevant formulas and carried out the calculations step by step to determine the height of the cuboid. The participant first multiplied the length and width of the cuboid $(12 \times 8 = 96)$, then divided the cuboid's volume by this result ($288 \div 96 = 3$) to obtain a height of 3 cm. However, an interesting pattern was observed in the participant's written work. Initially, the participant wrote the division operation in reverse $96 \div 288$ and then added a radical-like symbol beside the number ($9\sqrt{288}$) before finally writing the correct height of 3 cm. This finding suggests that participant GA1 focused more on following the solution procedures than on understanding the underlying mathematical concepts. The participant appeared to modify the mathematical symbols so that the written solution matched the expected final answer, even though the procedure was mathematically incorrect. This finding is consistent with Mubarik et al. (2026) who reported that students with an auditory learning style tend to show uncertainty in algebraic manipulation and limited confidence in generalizing problem-solving strategies. Despite these inaccuracies in the use of mathematical symbols, the participant's activity was still categorized as the Building-with stage because the participant successfully applied the solution procedures to completion, including calculating the volume of the cube using the recalled formula $3 \times 3 \times 3 = 27\ cm^3$.

At the Construction stage, participant GA1 did not meet the expected indicator. Based on the interview findings, the participant stated that no new understanding had been gained after completing the problem. When asked to explain the relationship between the cuboid and the cube using the drawing, the participant's explanation remained limited and referred only to the information provided in the problem. The participant also acknowledged that both the drawing and the solution procedure were based on explanations previously encountered on TikTok rather than on a self-developed understanding or strategy. Thus, the participant applied previously learned formulas without constructing a new conceptual understanding of the relationship between the two geometric solids. This finding is consistent with the study by Hadistian, F, (2023), which reported that not all students are able to reach the Construction stage in the RBC+C model. Some students are able to apply previously learned procedures during the Building-with stage but are not yet able to independently construct new conceptual understanding.

At the Consolidation stage, participant GA1 demonstrated a different pattern from the previous participant. Because the participant did not reach the Construction stage, the strengthening of understanding was primarily reflected in the Building-with stage. When asked about the effect of changing the dimensions of the cuboid, the participant explained that changes in the cuboid's dimensions would affect the volume of the cube because the side length of the cube depends on the height of the cuboid. The participant was also able to explain the solution procedure step by step using their own words, although a few minor errors occurred when recalling the formulas verbally.

At the end of the interview, the participant stated that they had gained a better understanding of how to determine the height of the cuboid and calculate the volume of the cube. The participant also expressed confidence in being able to solve similar problems in the future. These findings indicate that the solution procedures had been well understood and could be applied again when solving similar problems. However, this strengthened understanding remained limited to the application of solution procedures and did not demonstrate the development of new conceptual understanding, which is the defining characteristic of the Construction stage.

3.2.3 Error Analysis of Participant GK1

At the Recognizing stage, participant GK1 demonstrated a good understanding of the information presented in the problem. The participant successfully identified all the given information, including the volume, length, and width of the cuboid, as well as the condition that the side length of the cube was equal to the height of the cuboid. Before performing any calculations, the participant first planned the solution steps to make the problem-solving process more organized and systematic. Participant GK1 also accurately recalled the formulas for the volume of a cuboid and the volume of a cube $(s \times s \times s)$. As a participant with a kinesthetic learning style, GK1 related the problem to previous experience in solving similar exercises during earlier learning activities. This prior experience helped the participant recall the mathematical knowledge needed to solve the problem.

At the Building-with stage, participant GK1 successfully applied the relevant formulas and carried out the calculations in a logical, step-by-step manner. The participant first multiplied the length and width of the cuboid $(12 \times 8 = 96)$, then divided the cuboid's volume by this result $(288 \div 96)$ to obtain the height of the cuboid, which was 3 cm. Next, the participant used this value as the side length of the cube to calculate its volume $3 \times 3 \times 3 = 27\ cm^3$. Although the participant mistakenly referred to 96 as the height of the cuboid during the interview, whereas it actually represented the area of the cuboid's base, both the written work and the interview responses indicate that the participant understood the solution procedure well and was able to apply it systematically to obtain the correct answer.

At the Construction stage, participant GK1 successfully met the expected indicator. The participant not only applied the relevant formulas but also developed a new conceptual understanding of the relationship between the cuboid and the cube. Characteristics of the participant's kinesthetic learning style were evident in the explanation of the drawing that had been created. The participant explained that the height of the cuboid could be used as the side length of the cube to determine its volume. This indicates that the participant understood the relationship between the height of the cuboid and the volume of the cube. At the end of the problem-solving process, the participant also stated that a new understanding had been gained, namely that a dimension of the cuboid can be used to determine a corresponding dimension of the cube. This finding is consistent with the study by Budiarto et al. (2024) which states that the abstraction process develops when students are able to construct relationships among mathematical concepts through their learning experiences and visualizations.

At the Consolidation stage, participant GK1 demonstrated a stronger and more stable understanding of the concepts. When asked about the effect of changing the dimensions of the cuboid, the participant explained that such changes would also affect the volume of the cube because the two geometric solids are conceptually related. The participant was also able to briefly explain the solution procedure using their own words. In addition, the participant reviewed the completed work by rechecking the calculation steps that were still considered uncertain.

At the end of the interview, the participant stated that calculating the volume of the cube was the most challenging part of the task. Nevertheless, the participant reported having a better understanding of the relationship between the cuboid and the cube and expressed confidence in being able to solve similar problems in the future. This finding is consistent with the RBC+C model proposed by Hadistian, F, (2023), which suggests that students who reach the Construction stage demonstrate strengthened understanding during the Consolidation stage, enabling the knowledge they have constructed to be retained and applied when solving similar mathematical problems

4. CONCLUSION

This study concludes that ninth-grade students at SMP Negeri 14 Palu demonstrated diverse abstraction processes when solving polyhedral geometry problems involving cuboids and cubes based on the RBC+C model. The identified cognitive abilities included Recognizing, Building-with, Construction, and Consolidation, with the Construction stage emerging as the key factor distinguishing students' thinking processes. Differences among the participants were primarily reflected in the depth of the abstraction stages they achieved according to their learning styles rather than in their ability to apply mathematical formulas. The findings indicate that successful abstraction is closely related to students' ability to understand mathematical concepts and establish relationships among the concepts required to solve polyhedral geometry problems. Participants with visual and kinesthetic learning styles were able to construct new conceptual understanding of the relationship between cuboids and cubes, whereas the participant with an auditory learning style demonstrated consolidation mainly at the Building-with stage and did not reach the Construction stage.

Therefore, geometry instruction should consider students' learning style characteristics to better support the development of abstraction processes and conceptual understanding. This study has several limitations because it involved only three participants representing different learning styles and focused exclusively on polyhedral geometry involving cuboids and cubes. Consequently, the findings should be interpreted within the context of the participants and the mathematical topic investigated. Future research is recommended to involve a larger number of participants and different mathematics topics in order to provide a broader understanding of students' abstraction processes.

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