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PRESERVICE MATHEMATICS TEACHERS' CONCEPTUAL UNDERSTANDING OF LIMITS: A SYSTEMATIC REVIEW OF MATHEMATICAL AND PEDAGOGICAL KNOWLEDGE

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ABSTRACT

The concept of limits is a fundamental foundation of calculus; however, it remains one of the concepts that preservice mathematics teachers find difficult to understand conceptually. These difficulties are not limited to procedural problem-solving skills but also involve understanding the meaning of limits, coordinating multiple representations, constructing formal proofs, and developing the pedagogical ability to explain the concept effectively to students. This study aims to synthesize research on preservice mathematics teachers' conceptual understanding of the concept of limits by examining the dimensions of mathematical knowledge and pedagogical knowledge. A systematic literature review was conducted following the PRISMA 2020 guidelines. Data were collected from the Scopus and Dimensions databases through RIS files exported in June 2026. From an initial set of 127 records, 23 core articles were selected after the processes of deduplication and screening based on inclusion and exclusion criteria. The selected studies were analyzed using thematic synthesis. The review findings indicate that previous studies have predominantly focused on the mathematical knowledge dimension, particularly difficulties related to understanding the meaning of limits, transforming representations, graph visualization, formal definitions, mathematical proof, sequence limits, trigonometric limits, and asymptotic behavior. In contrast, the pedagogical knowledge dimension has received relatively less attention, although it has been explored through studies on anticipating students' responses, teacher noticing, representation selection, the use of GeoGebra, and technology-based didactical design. This review highlights that preservice mathematics teachers' conceptual understanding of limits should be viewed as an integration of mathematical knowledge and pedagogical knowledge. The findings imply the need for calculus instruction in teacher education programs that emphasizes multiple representations, mathematical proof, pedagogical reflection, analysis of students' thinking, and the conceptual integration of technology.

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1. INTRODUCTION

The concept of limits is one of the fundamental concepts in calculus, serving as the foundation for understanding derivatives, integrals, continuity, sequences, and advanced mathematical analysis. However, limits are not merely procedures involving substitution or algebraic manipulation; rather, they constitute a concept that requires an understanding of approximation processes, relationships between domains and codomains, symbolic representations, graphical representations, numerical representations, and formal definitions. For preservice mathematics teachers, conceptual understanding of limits is particularly important because they are expected not only to solve problems but also to explain the meaning of limits, anticipate students' misconceptions, and design instruction that supports the development of conceptual understanding.

Numerous studies have shown that preservice mathematics teachers continue to experience difficulties in understanding limits conceptually. Bansilal and Mkhwanazi (2021) found that preservice teachers often construct unstable meanings of limits because the concept is perceived as abstract and difficult to connect with formal representations. Similarly, Yıldız and İnceoğlu (2020) reported that preservice teachers' conceptual knowledge of limits of single-variable functions does not fully reflect an understanding consistent with formal mathematical definitions. In the Indonesian context, Nurafni (2016) demonstrated that cognitive style influences university students' understanding of limit concepts, while Mahmud and Takdirmin (2022) identified obstacles encountered by preservice teachers in visualizing limits of functions. These findings indicate that difficulties in understanding limits are not merely procedural in nature but are also related to how preservice teachers construct meaning, coordinate multiple representations, and interpret approximation processes.

These difficulties become even more complex when preservice teachers encounter more formal and diverse forms of limits. Amintoko et al. (2017) reported that mathematics education students experience cognitive obstacles in solving sequence limit problems, particularly in understanding formal definitions and visualizing real-number sequences. Tuna et al., (2019) found that preservice teachers' knowledge of sequence limits still exhibits conceptual weaknesses. Oktaviyanthi et al. (2018) demonstrated that proving function limits using formal definitions remains a challenge for preservice mathematics teachers. Furthermore, Mangwende & Chirume (2024), through the APOS framework, showed that preservice teachers' understanding of trigonometric function limits involves complex constructions of actions, processes, objects, and schemas. Therefore, understanding limits should be examined as a complex cognitive construction rather than merely as the outcome of mastering problem-solving algorithms.

Another important issue concerns the relationship between preservice teachers' mathematical understanding and pedagogical knowledge. From the perspective of pedagogical content knowledge, teachers must integrate content mastery with effective ways of representing and teaching that content to students (Shulman, 1986). In mathematics education, this requirement aligns with the concept of mathematical knowledge for teaching, which refers to the specialized mathematical knowledge needed to explain concepts, analyze students' responses, and select appropriate instructional representations (Ball et al., 2008). Kuzu (2010) showed that preservice teachers' competencies in transforming among representations of the limit concept constitute an essential component of conceptual understanding. Fernández et al., (2018) found that preservice teachers' ability to anticipate students' responses is related to coordinating approaches in both the domain and range of functions. More recently, Fernández et al. (2024) emphasized that noticing students' thinking about limits is a crucial aspect of the development of preservice teachers' pedagogical knowledge.

The advancement of educational technology has also broadened research on understanding limits. Zengin (2023) demonstrated that GeoGebra can help establish dynamic connections between formal definitions of function limits and sequence limits. Meika et al., (2025) showed that a TPACK-based e-didactics design can be used to address learning obstacles experienced by preservice mathematics teachers in differential calculus. These findings suggest that preservice teachers' understanding of limits is influenced not only by mathematical knowledge but also by representations, technology, didactical design, and pedagogical competence.

Although research on preservice mathematics teachers' understanding of limits has expanded considerably, existing studies remain scattered across topics such as visualization, representation, formal proof, APOS, GeoGebra, noticing, and TPACK. Most of these studies have not been systematically synthesized to explain how mathematical knowledge and pedagogical knowledge contribute to preservice teachers' conceptual understanding of limits. This gap is important because preservice mathematics teachers must not only understand limits as mathematical objects but also be capable of transforming such understanding into teachable knowledge.

In response to this gap, the present study aims to conduct a systematic review of preservice mathematics teachers' conceptual understanding of the concept of limits by examining the dimensions of mathematical knowledge and pedagogical knowledge. This review is expected to map current research trends, identify forms of conceptual difficulties experienced by preservice teachers, analyze the roles of representations and technology, and propose directions for future research in mathematics teacher education.

2. METHOD

This study employed a systematic literature review (SLR) to synthesize empirical findings concerning preservice mathematics teachers' conceptual understanding of the concept of limits, with particular emphasis on the dimensions of mathematical knowledge and pedagogical knowledge. A systematic review approach was selected because research on limits among preservice mathematics teachers has been distributed across diverse themes, including representation, visualization, formal proof, APOS theory, technology integration, noticing, and didactical design. Consequently, this method enables researchers to identify patterns in the literature, reveal research gaps, and propose future research directions in a transparent and structured manner. The review was reported in accordance with the PRISMA 2020 guidelines, which emphasize transparency in the processes of identification, selection, appraisal, and synthesis of studies (Page et al., 2021). Furthermore, the literature search strategy was developed following the principles of PRISMA-S to ensure methodological transparency and replicability (Rethlefsen et al., 2021).

Data Sources

The data sources for this study were the Scopus and Dimensions databases. These databases were selected because they include both international and national scholarly publications relevant to mathematics education, teacher education, calculus, and the teaching and learning of limits. The initial dataset was obtained from RIS files exported on June 1, 2026. Based on metadata examination, the preliminary search yielded 127 records, consisting of 18 records from Scopus, 7 additional Scopus records, and 102 records from Dimensions. After duplicate removal based on DOI and article title, 120 unique records remained. The final number of articles included in the review was determined through title, abstract, and full-text screening according to predefined inclusion and exclusion criteria.

Search Strategy

The search strategy was developed by combining keywords representing three major components: the study population, the mathematical concept, and the knowledge focus. The keywords included *"preservice mathematics teacher,"* "pre-service mathematics teacher*," "prospective mathematics teacher*," "mathematics teacher candidate*," "calon guru matematika," "limit concept," "limit function," "limits of sequences," "conceptual understanding," "mathematical knowledge," "pedagogical knowledge," "pedagogical content knowledge," "representation," "visualization," "proof," "APOS," "GeoGebra,"* and *"TPACK."* Search strings were adapted to the syntax requirements of each database. In general, the search query was formulated as follows:

("preservice mathematics teacher*" OR "pre-service mathematics teacher*" OR "prospective mathematics teacher*" OR "mathematics teacher candidate*" OR "calon guru matematika") AND ("limit concept" OR "limit function" OR "limits of sequences" OR "limit trigonometric function") AND ("conceptual understanding" OR "mathematical knowledge" OR "pedagogical knowledge" OR "representation" OR "visualization" OR "proof" OR "APOS" OR "GeoGebra" OR "TPACK").

No restriction was imposed on the starting publication year in order to capture the historical development of research in this area. The search period was limited to publications available up to June 2026.

Inclusion and Exclusion Criteria

The inclusion criteria consisted of five requirements. First, the study had to involve preservice mathematics teachers, mathematics education students, or prospective mathematics teachers. Second, the study had to focus on the concept of limits, including function limits, sequence limits, trigonometric limits, or formal definitions of limits. Third, the study had to address conceptual understanding, representation, visualization, proof, misconceptions, learning obstacles, or pedagogical knowledge related to limits. Fourth, the article had to report empirical research using qualitative, quantitative, mixed-methods, or didactical design approaches. Fifth, the article had to be published in either English or Indonesian and provide sufficient bibliographic information.

Articles were excluded if they focused on calculus in general without explicitly addressing limits, examined school students without any connection to preservice mathematics teachers, consisted of editorials or conference proceedings lacking a clear scholarly review process, were duplicate records, or did not provide access to an abstract and adequate methodological information.

Study Selection Process

The selection process was conducted in three stages. The first stage involved duplicate checking based on DOI, title, author names, and publication year. The second stage consisted of title and abstract screening to determine the relevance of each article to the review focus. The third stage involved full-text assessment to establish final eligibility. Each article was coded as *included*, *excluded*, or *requiring further discussion*. Reasons for exclusion at the full-text stage were documented, such as lack of focus on limits, absence of preservice mathematics teacher participants, or insufficient empirical evidence. The selection procedure was summarized using a PRISMA flow diagram to ensure transparency and traceability throughout the review process.

Data Extraction

Data from the selected studies were extracted using a structured extraction matrix. The matrix included article identification, country or research context, research objectives, study design, participants, type of limit concept investigated, theoretical framework, instruments, data analysis techniques, main findings, and implications for mathematics teacher education.

In addition, data extraction focused on two primary conceptual categories: **mathematical knowledge** and **pedagogical knowledge**. The mathematical knowledge category included understanding of formal definitions, symbolic representations, graphical representations, numerical representations, visualization, proof, coordination of approximation processes, and cognitive constructions such as actions, processes, objects, and schemas. The pedagogical knowledge category included the ability to explain concepts, select appropriate representations, anticipate students' responses, identify misconceptions, engage in noticing, and utilize technology and didactical design in teaching limits. These categories were informed by the frameworks of pedagogical content knowledge (Shulman, 1986) and mathematical knowledge for teaching (Ball et al., 2008).

Quality Assessment

The methodological quality of the included studies was evaluated using an appraisal framework adapted to the characteristics of each research design. The assessment criteria included clarity of research objectives, appropriateness of the research design, adequacy of participant and context descriptions, clarity of instruments or data sources, appropriateness of data analysis procedures, traceability of findings to the data, and relevance of the reported implications.

Studies were not automatically excluded based on methodological limitations; instead, quality assessments were used to evaluate the strength of evidence during the synthesis process. This approach was considered appropriate because research on preservice mathematics teachers' understanding of limits is highly heterogeneous in terms of design, context, and data characteristics.

Data Analysis

The data were analyzed using thematic synthesis. The analysis followed the principles of thematic analysis proposed by Braun & Clarke (2008), including repeated reading, initial coding, code grouping, theme development, theme review, and synthesis writing. Both deductive and inductive approaches were employed.

The deductive analysis was guided by the two predefined categories of **mathematical knowledge** and **pedagogical knowledge**, whereas the inductive analysis was used to identify emerging themes across the studies, such as misconceptions about limits, difficulties with formal proof, the role of representations, the use of GeoGebra, learning obstacles, APOS theory, and noticing students' thinking.

To ensure analytical consistency, each code was repeatedly checked against the findings reported in the original studies and compared with the objectives of the review. The synthesized findings were then presented in the form of thematic narratives, tables describing article characteristics, mappings of research foci, and summaries of identified research gaps.

Trustworthiness of the Review

The reliability of the review process was maintained through comprehensive documentation of all stages, including searching, screening, data extraction, and coding. The researchers preserved search strings, search dates, record counts, exclusion reasons, and data extraction matrices. The validity of the synthesis was strengthened by comparing the emerging themes with the theoretical framework outlined in the introduction, particularly the relationship among conceptual understanding of limits, mathematical knowledge, and pedagogical knowledge in preservice mathematics teachers.

Through these procedures, the present study aims to provide a transparent, replicable, and meaningful synthesis that contributes to the advancement of mathematics teacher education research.

3. RESULTS AND DISCUSSION

3.1. Results

The initial search conducted through the Scopus and Dimensions databases yielded 127 records. These records consisted of 102 articles retrieved from Dimensions and 25 articles obtained from two Scopus export files. Following duplicate screening based on DOI and article title, seven duplicate records were identified and removed, resulting in 120 unique records for further screening.

Title and abstract screening identified 34 articles that were potentially relevant to the focus of this review, namely the conceptual understanding of limits among preservice mathematics teachers, mathematics education students, or prospective mathematics teachers.

Of these 34 articles, 11 were excluded because they did not focus primarily on the concept of limits, addressed calculus in general, examined other mathematical topics such as continuity, general proof, problem posing, or arithmetic visualization, or involved preservice teachers from disciplines other than mathematics. Consequently, 23 core articles met the inclusion criteria and were selected for further analysis. Among these 23 articles, 22 included a DOI in their RIS metadata, while one article did not provide DOI information.

General Characteristics of the Selected Studies

The selected studies were published between 2016 and 2025. The publication trend indicates a gradual increase in research on preservice mathematics teachers' understanding of limits during recent years. The earliest study included in the corpus was conducted by Nurafni (2016), which examined students' understanding of limit concepts from the perspective of cognitive style. The highest number of publications appeared in 2025, with six studies addressing topics such as the negation of formal limit definitions, misconceptions about function limits, conceptual understanding viewed through mathematical beliefs, sequence limits, GeoGebra-based augmented reality media, and trigonometric function limits (Ananda et al., 2025; Hajerina et al., 2025; Mangwende & Chirume, 2024; Simamora et al., 2025; Usman & Hasbi, 2025; Veronika et al., 2025).

Regarding publication sources, the selected studies originated from both national and international mathematics education journals. National studies predominantly examined mathematics education students in Indonesian universities, including UHAMKA, Universitas Hasyim Asy'ari, Universitas Ahmad Dahlan, Universitas Negeri Medan, Universitas Muhammadiyah Makassar, Universitas Muhammadiyah Mataram, and Alkhairaat University. International studies investigated preservice mathematics teachers in broader contexts, including Turkey, South Africa, Spain, and other international mathematics teacher education settings (Bansilal & Mkhwanazi, 2021; Fernández et al., 2018; Kuzu, 2010; Nurafni, 2016; Tuna et al., 2019; Yıldız & İnceoğlu, 2020).

Focus Areas of the Limit Concept

Based on content coverage, the selected studies can be classified into five major areas.

First, most studies focused on function limits, particularly in relation to conceptual understanding, visualization, representation, and students' errors. This category includes studies by Nurafni (2016), Mahmud and Takdirmin (2022), Yıldız and İnceoğlu (2020), Bansilal and Mkhwanazi (2021), Usman et al. (2025), and Hajerina et al. (2025).

Second, several studies examined formal definitions and proofs of function limits. This focus is evident in the studies of Oktaviyanthi et al. (2018), Qadry et al. (2021), Simamora et al. (2025), and Sumargiyani et al. (2021). These studies treated the formal definition of limits as a mathematical object requiring symbolic competence, logical reasoning, quantifier usage, and proof construction.

Third, some studies investigated sequence limits. Amintoko et al. (2017) explored students' cognitive obstacles in solving sequence limit problems and the role of scaffolding in overcoming these difficulties. Tuna et al. (2019) examined preservice teachers' knowledge of sequence limits, while Veronika et al. (2025) analyzed students' errors in solving sequence and sequence-limit problems using Newman's error analysis procedure.

Fourth, several studies addressed more specialized forms of limits, including trigonometric function limits and asymptotic behavior. Mangwende and Chirume (2025) employed the APOS framework to analyze preservice mathematics teachers' understanding of trigonometric function limits. Similarly, Katalenić et al. (2023) investigated preservice teachers' knowledge of asymptotes and asymptotic behavior in calculus, both of which are closely related to the concept of limits.

Fifth, a number of studies considered limits within the context of representations and educational technology. Kuzu, (2010) investigated transformations among representations of the limit concept. Zengin (2023) examined the use of GeoGebra for establishing connections between formal definitions of function limits and sequence limits. Likewise, Ananda et al. (2025) explored the use of GeoGebra-based augmented reality media to facilitate students' conceptual understanding of function limits.

Findings Related to Mathematical Knowledge

The synthesis revealed that the dimension of mathematical knowledge was the most dominant theme among the selected studies. The most frequently reported issue concerned weaknesses in preservice mathematics teachers' conceptual understanding of the meaning of limits. Nurafni (2016) found that students with a field-dependent cognitive style tended to focus on limit notation without successfully connecting given information to relevant conceptual ideas. Bansilal and Mkhwanazi (2022) reported that preservice teachers experienced difficulties distinguishing between the necessary conditions and conceptual constructs underlying the existence of limits. Similarly, Yıldız and İnceoğlu (2020) found that preservice teachers often relied on memorized responses and tended to avoid formal definitions of limits. Usman et al. (2025) further demonstrated that students with moderate and low mathematical beliefs were more likely to depend on procedural approaches rather than conceptual explanations.

A second recurring theme involved difficulties related to representation and visualization. Mahmud and Takdirmin (2022) reported that preservice mathematics teachers struggled to sketch graphs accurately and to visualize situations in which limits do not exist. Kuzu (2010) found that preservice teachers encountered challenges in transforming among representations, particularly in interpreting verbal representations and understanding limit points. Furthermore, Ananda et al. (2025) showed that GeoGebra-based augmented reality media helped students visualize cluster points, formal definitions of limits, and one-sided limits.

A third theme concerned difficulties with formal definitions, negation, and proof. Oktaviyanthi et al. (2018) identified two major stages in proving limits formally: proof preparation and proof construction. Qadry et al. (2021) found that students experienced difficulties negating formal definitions of limits, particularly statements involving multiple quantifiers. Simamora et al. (2025) reported errors in reading, comprehension, transformation, process execution, and response formulation when students were asked to negate the formal definition of a function limit. Similarly, Sumargiyani et al. (2021) found that many preservice teachers lacked sufficient understanding of one-sided limits and experienced difficulties using mathematical symbols correctly in proof-related tasks.

A fourth theme involved limitations in understanding sequence limits, trigonometric limits, and asymptotic behavior. Amintoko et al. (2017) found that students' difficulties with sequence limits were associated with insufficient prior knowledge, weak analogical reasoning, and limited mathematical connections. Tuna et al. (2019) identified weaknesses in preservice teachers' understanding of accumulation points in sequence limits. Mangwende and Chirume (2025) reported procedural, conceptual, and extrapolation errors when preservice teachers applied sine limit identities to trigonometric function limits. Likewise, Katalenić et al. (2023) found that preservice teachers' understanding of asymptotes and asymptotic behavior was fragmented and heavily dependent on memorized formulas and algebraic manipulation.

Findings Related to Pedagogical Knowledge

The dimension of pedagogical knowledge emerged primarily in studies addressing instructional representations, anticipation of students' responses, noticing, and technology integration. Fernández et al. (2018) found that preservice teachers who were able to identify coordinated approaches in the domain and range of functions could use this knowledge to evaluate students' conceptual development and make informed instructional decisions. Fernández et al. (2024) further demonstrated that preservice teachers' noticing of students' thinking about limits evolved from a dichotomous perspective toward a more developmental and conceptual understanding.

Studies focusing on instructional representations revealed that preservice teachers employed multiple forms of representation when teaching limits. Ünver and Güzel (2019) identified the use of verbal explanations, number lines, tables, figures, graphs, and algebraic representations. However, verbal representations were used most frequently, whereas number-line representations were used least often. Similarly, Kuzu (2020) found that transformation among representations remained challenging, particularly when problems were presented verbally.

Technology-based studies suggested that GeoGebra and other interactive media can support visualization and conceptual connections. Zengin (2023) reported that collaborative learning activities using GeoGebra facilitated the development of dynamic connections among accumulation points, sequence limits, and function limits. Ananda et al. (2025) found that interactive features such as sliders in GeoGebra helped students understand changes in limit values and increased their engagement in learning activities. In addition, Abdillah (2017) demonstrated that elaboration strategies could be effectively employed in teaching function limits through stages of orientation, interpretation, and conclusion.

Summary of Thematic Mapping

Table 1. Thematic Mapping of Research on Preservice Mathematics Teachers' Understanding of Limits

Theme	Main Focus	Related Studies
Conceptual Understanding of Limits	Meaning of limits, procedural versus conceptual understanding, and misconceptions	Nurafni (2016); Bansilal and Mkhwanazi (2022); Yıldız and İnceoğlu (2020); Usman et al. (2025); Hajerina et al. (2025)
Representation and Visualization	Graphical, verbal, numerical, and algebraic representations; transformation among representations	Mahmud and Takdirmin (2022); Kuzu (2020); Ünver and Güzel (2019); Ananda et al. (2025); Zengin (2023)
Formal Definitions and Proof	Formal proof of limits, negation of limit definitions, quantifiers, and symbolic reasoning	Oktaviyanthi et al. (2018); Qadry et al. (2021); Simamora et al. (2025); Sumargiyani et al. (2021)
Sequence Limits and Specialized Forms of Limits	Sequence limits, trigonometric limits, and asymptotic behavior	Amintoko et al. (2017); Tuna et al. (2019); Veronika et al. (2025); Mangwende and Chirume (2025); Katalenić et al. (2023)
Pedagogical Knowledge	Anticipation of students' responses, teacher noticing, selection of representations, and instructional strategies	Fernández et al. (2018); Fernández et al. (2024); Ünver and Güzel (2019); Abdillah (2017); Zengin (2023)

Source: Synthesized from the selected studies included in this review.

Overall, the review demonstrates that the selected studies predominantly frame limits as an issue of mathematical knowledge, particularly regarding conceptual understanding, representational difficulties, formal proof, and misconceptions. Research explicitly addressing limits within the framework of pedagogical knowledge is less common, although it appears in studies focusing on students' responses, noticing, representational choices, and educational technology.

3.2 Discussion

The findings of this review indicate that preservice mathematics teachers' conceptual understanding of limits remains a fundamental challenge in calculus education. One of the most prominent findings is the dominance of difficulties associated with **mathematical knowledge**, particularly in understanding the meaning of limits, coordinating multiple representations, interpreting approximation processes, applying formal definitions, and distinguishing procedural understanding from conceptual understanding. These findings suggest that limits are often treated as computational procedures rather than as mathematical objects with complex conceptual and epistemological structures. The studies conducted by Nurafni (2016), Bansilal and Mkhwanazi (2022), Yıldız and İnceoğlu (2020), Hajerina et al. (2025), and Usman et al. (2025) consistently demonstrate that preservice teachers' difficulties extend beyond computation to include explaining concepts, connecting representations, and providing mathematical justifications for the existence or nonexistence of limits.

This finding is particularly significant because preservice mathematics teachers occupy a dual role as learners of mathematics and future educators. In this context, an incomplete conceptual understanding of limits may directly influence the quality of instruction they provide in the future. When preservice teachers understand limits primarily through substitution, factorization, or algebraic manipulation, they may unintentionally promote procedural approaches in their own teaching. This observation supports the notion of *mathematical knowledge for teaching*, which emphasizes that teachers require more than the ability to solve problems; they must also be capable of explaining mathematical ideas, evaluating students' responses, selecting appropriate representations, and fostering conceptual connections (Ball et al., 2008).

Another important pattern emerging from this review concerns difficulties in coordinating multiple representations. Mahmud and Takdirmin (2022) found that preservice teachers experienced obstacles in sketching graphs and visualizing situations in which limits do not exist. Similarly, Kuzu (2020) reported that transforming among verbal, graphical, numerical, and symbolic representations remains challenging. These findings indicate that understanding limits requires strong representational competence. Limits cannot be fully understood through symbolic expressions alone; rather, they require the integration of function values, graphical behavior, numerical patterns, one-sided approaches, and formal conditions. Therefore, representations should be viewed not merely as instructional tools but as essential components of conceptual understanding itself.

The difficulties associated with formal definitions and mathematical proof reveal a deeper layer of conceptual challenges. Studies by Oktaviyanthi et al. (2018), Qadry et al. (2021), Simamora et al. (2025), and Sumargiyani et al. (2021) show that preservice teachers often struggle with epsilon–delta definitions, logical quantifiers, negations, proof construction, and mathematical argumentation. These findings highlight a substantial gap between intuitive and formal understanding. While students may successfully solve routine limit problems, they frequently encounter difficulties when required to explain why a limit exists, justify mathematical statements formally, or construct rigorous proofs. This gap represents a critical issue in teacher education because effective mathematics instruction requires teachers who can bridge students' intuitive reasoning and formal mathematical thinking.

The review also demonstrates that sequence limits, trigonometric limits, and asymptotic behavior introduce additional layers of complexity. Amintoko et al. (2017) and Tuna et al. (2019) found that understanding sequence limits requires knowledge of patterns, convergence processes, accumulation points, and the visualization of real-number sequences. Furthermore, Mangwende and Chirume (2025) showed that understanding trigonometric limits involves cognitive constructions that can be explained through the APOS framework, including actions, processes, objects, and schemas. Similarly, Katalenić et al., (2023) reported that preservice teachers' understanding of asymptotes and asymptotic behavior is often fragmented and heavily dependent on memorized procedures. Collectively, these findings suggest that limits should not be viewed as a single mathematical procedure but rather as a concept that appears in diverse mathematical contexts requiring flexible and interconnected thinking.

In contrast to the extensive focus on mathematical knowledge, the **pedagogical knowledge** dimension received relatively limited attention in the reviewed studies. Research explicitly addressing pedagogical knowledge primarily focused on anticipating students' responses, teacher noticing, representational choices, and the integration of technology into instruction. Fernández et al., (2018) demonstrated that preservice teachers' ability to anticipate students' responses is closely related to their understanding of coordinated approaches within the domain and range of functions. Likewise, Fernández et al. (2024) found that preservice teachers' noticing of students' thinking evolved from a dichotomous perspective toward a more developmental and conceptual interpretation. These findings suggest that pedagogical understanding of limits extends beyond content mastery and requires the ability to interpret students' reasoning, identify misconceptions, and design effective instructional responses.

From the perspective of **Pedagogical Content Knowledge (PCK)**, teachers must integrate subject matter knowledge with strategies for teaching that knowledge effectively (Shulman, 1986). Consequently, the findings of this review suggest that research on preservice mathematics teachers should not be limited to evaluating whether they can correctly solve limit problems. Instead, greater attention should be devoted to examining how they explain limits, select examples and non-examples, interpret students' errors, utilize graphs and tables, connect intuitive reasoning with formal definitions, and design learning activities that promote conceptual understanding. In this sense, conceptual understanding of limits should be viewed as the intersection between mathematical knowledge and pedagogical knowledge.

The integration of technology, particularly GeoGebra and other interactive learning environments, emerged as a promising theme. Zengin (2023) demonstrated that GeoGebra can support the development of dynamic connections between function limits and sequence limits. Similarly, Ananda et al., (2025) reported that GeoGebra-based augmented reality media facilitates the visualization of limit concepts and enhances student engagement. Furthermore, Meika et al. (2025) found that TPACK-based e-didactics designs can help address learning obstacles in differential calculus. Nevertheless, these findings should be interpreted cautiously. Technology alone does not guarantee conceptual understanding unless it is accompanied by meaningful learning tasks, reflective questioning, mathematical discussion, and explicit connections among representations. Therefore, technology integration should be directed toward meaning-making rather than merely providing attractive visualizations.

The primary contribution of this review lies in demonstrating that previous studies on preservice mathematics teachers' understanding of limits have largely concentrated on mathematical knowledge, whereas pedagogical knowledge has received comparatively less explicit attention. The novelty of this review is its attempt to synthesize these two dimensions within a single framework, highlighting how preservice teachers understand limits as mathematical objects and how that understanding can be transformed into teachable knowledge. This synthesis suggests that difficulties with limits should be viewed not only as issues of calculus learning but also as broader challenges in mathematics teacher education.

Theoretically, the findings point to the need for a conceptual model that integrates **conceptual understanding of limits**, **mathematical knowledge**, and **pedagogical knowledge**. Such a model may consist of three interconnected layers. The first layer involves mathematical understanding, including the meaning of limits, representations, formal definitions, and proof. The second layer involves pedagogical understanding, including the ability to explain concepts, select representations, anticipate students' responses, and identify misconceptions. The third layer involves didactical integration, including the use of technology, instructional task design, and teaching strategies that support students' conceptual development.

From a practical perspective, mathematics teacher education programs should strengthen calculus instruction through approaches that emphasize representations, proof, pedagogical reflection, and the analysis of students' thinking. The teaching of limits should move beyond routine procedural exercises and incorporate tasks that require students to explain the meaning of limits in their own words, compare graphical and symbolic representations, evaluate mathematical arguments, negate formal definitions, and design explanations appropriate for secondary school students. In addition, the use of GeoGebra, augmented reality, and TPACK-based instructional designs should be directed toward helping preservice teachers connect intuition, visualization, and mathematical formalization.

Several limitations should be acknowledged. This review relied exclusively on studies indexed in the Scopus and Dimensions databases, which may have excluded relevant research published elsewhere. Furthermore, several national studies focused on highly specific contexts, limiting the generalizability of the findings. The diversity of research designs, participant characteristics, instruments, and theoretical frameworks also restricted the possibility of conducting a quantitative meta-analysis. Therefore, the present study should be regarded as a thematic synthesis rather than a statistical meta-analysis.

Future research should focus on four major directions. First, more comprehensive instruments are needed to assess conceptual understanding of limits, including representations, formal definitions, proof, and pedagogical knowledge. Second, future studies should examine the relationships among mathematical beliefs, conceptual understanding, and teaching competence related to limits. Third, technology-based interventions should be evaluated not only in terms of motivation and visualization but also in terms of their effectiveness in supporting conceptual and pedagogical understanding. Finally, longitudinal studies are needed to investigate how preservice teachers' understanding of limits develops from undergraduate calculus courses to actual teaching practice. Such efforts will contribute to a more comprehensive understanding of limits within mathematics teacher education and support the preparation of future mathematics teachers.

4. CONCLUSION

This study demonstrates that preservice mathematics teachers' conceptual understanding of the concept of limits remains an important issue in both calculus education and mathematics teacher education. Based on the synthesis of 23 core articles, the difficulties experienced by preservice mathematics teachers are not limited to solving limit problems procedurally. They also involve understanding the meaning of limits as a process of approaching a value, coordinating symbolic, numerical, graphical, and verbal representations, and applying formal definitions appropriately. These difficulties are evident across various contexts, including function limits, sequence limits, trigonometric function limits, formal proofs, the negation of limit definitions, and asymptotic behavior.

The main findings of this review indicate that previous studies have predominantly focused on the dimension of **mathematical knowledge**. Most of the reviewed articles examined misconceptions, representational errors, visualization difficulties, weaknesses in mathematical proof, and limited understanding of the formal structure of the limit concept. In contrast, the dimension of **pedagogical knowledge** has received comparatively less explicit attention. Pedagogical aspects were mainly explored through studies on anticipating students' responses, teacher noticing, the selection of instructional representations, elaboration strategies, the use of GeoGebra, Augmented Reality, and TPACK-based e-didactics designs.

Therefore, the primary contribution of this study is to emphasize that preservice mathematics teachers' conceptual understanding of limits should be viewed as an integration of mathematical knowledge and pedagogical knowledge. Preservice teachers need more than the ability to calculate limits correctly; they must also be able to explain the meaning of limits, select appropriate representations, identify students' misconceptions, connect intuition with formal definitions, and design instructional activities that promote conceptual understanding.

The findings imply that mathematics teacher education programs should strengthen calculus instruction through approaches that are more conceptual, representational, reflective, and pedagogically oriented. The teaching of limits should include the exploration of graphs, tables of values, formal proofs, error analysis, discussions of students' thinking, and the purposeful use of technologies such as GeoGebra. Future research should develop instruments capable of assessing the integration of mathematical knowledge and pedagogical knowledge in relation to the concept of limits and investigate instructional designs that can support the transition from procedural understanding to deeper conceptual and pedagogical understanding.

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