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PROBLEM-BASED LEARNING WITHIN UBD FRAMEWORK TO ENHANCE MATHEMATICAL CONCEPTUAL UNDERSTANDING OF CIRCLE EQUATION

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ABSTRACT

This study aims to examine the effectiveness of implementing Problem-Based Learning within the Understanding by Design framework in enhancing students' conceptual understanding of circle equations. This study employs a mixed-methods approach using a Sequential Explanatory Design. The subjects were 30 eleventh-grade students at SMAN 4 Malang. Quantitative data were collected through pre-test and post-test, while qualitative data were gathered through questionnaires, observations, interviews, and documentation. The results showed a significant improvement in students' conceptual understanding, with the mean score increasing from 6.06 (pre-test) to 8.21 (post-test). The paired-sample t-test revealed a statistically significant difference with a large effect size (Cohen's d = 1.131). Questionnaire results indicated that students responded positively to the implementation, particularly appreciating the clarity of learning objectives, the alignment between assessments and instruction, the use of contextual problems, and the support of visual media and collaborative activities. This study provides empirical evidence that integrating the principles of backward design in UbD with the PBL syntax can serve as an effective alternative approach to developing students' conceptual understanding of mathematics.

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1. INTRODUCTION

Conceptual understanding is one of the primary objectives of mathematics education, as it enables students to understand the relationships between concepts, explain the reasons behind mathematical procedures, and apply their knowledge to a variety of new situations. However, various studies indicate that students still tend to master problem-solving procedures without gaining a deep understanding of the underlying concepts (Payadnya et al., 2023). This problem is frequently encountered in analytical geometry, particularly with circle equations, which require the ability to integrate concepts of geometry, coordinate systems, and the Pythagorean Theorem into a unified understanding. Kpotosu et al., (2024) and Niam et al. (2024) report that students often struggle to connect geometric representations with algebraic forms when solving problems related to circles.

Diagnostic assessment results conducted prior to this study indicate that students' conceptual understanding of circle equations remains low. Most students were able to apply the given formulae, but struggled when asked to explain the meaning of the elements in a circle equation, to identify the relationship between the centre and the radius and the form of the equation, and to construct a circle equation based on the given geometric representation. These findings indicate that the current teaching has not yet fully helped students to develop meaningful conceptual understanding.

This low level of conceptual understanding is not only due to the complexity of the subject matter, but is also linked to the teaching approach employed. Mathematics teaching, which remains dominated by lecture-based and teacher-centred methods, tends to position students as passive recipients of information, thereby limiting their involvement in the process of knowledge construction (Kalita & Nath, 2025; Lee & Paul, 2023). Furthermore, the diverse characteristics of students demand learning strategies capable of accommodating different learning needs. Research by Saputra et al. (2021) indicates that students' learning characteristics influence the level of conceptual understanding achieved. Meanwhile, students with visual and kinesthetic learning tendencies require concrete representations, visualisations, and exploratory activities to understand abstract mathematical concepts (Laurentino et al., 2024; Yurniwati, 2018). When teaching focuses solely on verbal explanations and symbolic manipulation, students often struggle to construct a complete mental representation of the concepts being studied.

One learning model considered capable of addressing these issues is Problem-Based Learning (PBL). This model uses contextual problems as the starting point for learning, thereby encouraging students to investigate, discuss and develop their understanding through the problem-solving process (Arends, 2012; Hidayat & Fitrini, 2025; Peranginangin, 2025). In the context of circle equations, PBL has the potential to help students make more meaningful connections between geometric, visual, and algebraic representations. Research by Mapeni et al. (2026) indicates that PBL is more effective than conventional teaching in improving students' mathematical abilities and is capable of accommodating diverse learning characteristics. Furthermore Payadnya et al. (2023) found that problem-solving activities based on exploratory questions can encourage students to think critically and creatively when learning geometric concepts.

Nevertheless, the effectiveness of PBL depends heavily on the quality of the instructional design. Therefore, an instructional design framework is required that can ensure alignment between learning objectives, learning activities, and assessment. Understanding by Design (UbD), developed by Wiggins & McTighe (2005). offers a backward design approach through three main stages: determining the expected learning outcomes, defining evidence of achievement, and designing appropriate learning experiences. This approach is believed to address the misalignment between learning objectives, processes, and evaluation often found in conventional teaching practices. According to Liu (2026), UbD provides a systematic framework that integrates content analysis, learning objectives, assessment, and learning activities, thereby supporting the creation of learning focused on deep understanding. Findings by Ostinelli (2024) also indicate that UbD-based mathematics learning can enhance student engagement and help them build a more organised knowledge structure. Furthermore, Aristanti & Fatayan (2024) report that the implementation of PBL designed according to UbD principles has a positive impact on students' critical thinking and problem-solving skills.

Previous studies indicate that PBL and UbD have each been extensively researched in mathematics education. However, research that explicitly integrates these two approaches to enhance students' conceptual understanding of circle equations remains limited. Most previous research has focused on improving problem-solving skills, critical thinking, or mathematical connections (Aristanti & Fatayan, 2024; Niam et al., 2024), while studies emphasising conceptual understanding of the topic of equations of circles have not been widely found. Therefore, the novelty of this study lies in the integration of Problem-Based Learning and Understanding by Design within a learning design specifically aimed at developing students' conceptual understanding of the topic of circle equations.

Based on the above, this study aims to examine the effectiveness of implementing Problem-Based Learning within the Understanding by Design framework in enhancing students' conceptual understanding of circle equations. The findings are expected to make a theoretical contribution to the development of mathematics instructional design and to provide a practical alternative for teachers in designing more meaningful, concept-oriented lessons.

2. METHOD

This study employs a mixed-methods approach using a Sequential Explanatory Design, which is a research design that begins with the collection and analysis of quantitative data, followed by the collection and analysis of qualitative data to explain and elaborate on the quantitative findings (Creswell & Plano Clark, 2018). In the quantitative research phase, the study aimed to examine the effect of implementing Problem-Based Learning (PBL) within the Understanding by Design (UbD) framework on students' conceptual understanding of circle equations. Subsequently, the qualitative stage was conducted to explore the learning experiences, responses, and difficulties encountered by students during the learning process. The integration of the two types of data was carried out at the interpretation stage through a 'connecting' strategy, namely using the quantitative results as the basis for participant selection and the focus of exploration in the qualitative stage (Creswell & Plano Clark, 2018).

The study was conducted at SMAN 4 Malang, Indonesia, during the second semester of the 2025/2026 academic year. The school was selected using purposive sampling because eleventh-grade students were studying the topic of circle equations, which aligned with the research objectives. The subjects of the quantitative study comprised all 30 eleventh-grade students, aged between 16 and 17 years. Total sampling was used to ensure that all students participated in the learning activities and in the assessment of their initial and final abilities. In the qualitative stage, three students were selected using purposive sampling based on the results of the post-test, representing the high, medium and low ability categories. The selection of these three students was intended to provide a more comprehensive picture of the students' learning experiences during the implementation of the learning process.

Research data were collected through tests, questionnaires, observations, interviews and documentation. The test instruments consisted of pre-tests and post-tests designed to measure students' conceptual understanding of the material on equations of circles. The development of the instruments referred to indicators of conceptual understanding in mathematics, which included: (1) explaining the concepts and definitions of circle equations; (2) identifying the relationship between the centre, radius, and form of circle equations; (3) determining circle equations based on the geometric information provided; and (4) interpreting geometric and algebraic representations in an integrated manner. The test instrument blueprint is presented in table 1.

Table 1. Blueprint of the Conceptual Understanding Test Instrument

Conceptual Understanding Indicators	Question Number
Explaining the concepts and definitions of circle equations	1–2
Relating the centre and the radius using the equation of a circle	3–4
Determining the equation of a circle from given geometric representation	5–6
Interpreting geometric and algebraic representations of the circle equation's concept	7–8

In addition to the tests, the study utilised a questionnaire to examine students' responses to the implementation of PBL within the UbD framework, comprising 10 statements on a four-point Likert scale, ranging from 'strongly disagree' to 'strongly agree'. Structured observations were carried out to observe student activities and the implementation of learning, while semi-structured interviews were used to explore students' learning experiences and the difficulties they faced. Documentation in the form of photographs of activities, learning materials, student work, and field notes was used as supporting data.

All research instruments were validated through expert judgement involving two validators, one mathematics education lecturers and one experienced mathematics teacher. The test instruments were then pilot-tested on 34 students who had studied the topic of circle equation. Item validity was analysed using Pearson's correlation with the aid of SPSS version 22. All items had a validity coefficient greater than 0.30 and were therefore deemed valid for use in the study (Hair Jr et al., 2019). The results of the validity analysis showed that the coefficient values ranged from 0.42 to 0.66. The reliability test using Cronbach's Alpha coefficient yielded a value of 0.68, indicating sufficient internal consistency and remaining acceptable for exploratory educational research with a limited number of items (Taber, 2018).

Before the study was conducted, the researcher obtained permission from the school and ensured that all participants took part in the study voluntarily. The confidentiality of the students' identities was maintained throughout the data collection and reporting process.

The research procedure followed the Sequential Explanatory Design framework. In the quantitative phase, students were first given a pre-test to measure their initial abilities. Subsequently, teaching took place over two sessions, 3×45 minutes in the first session and 2×45 minutes in the second session, using the PBL model integrated within the UbD framework. The UbD stage involves: (1) determining the desired results; (2) designing assessment evidence; and (3) developing a learning plan (Wiggins & McTighe, 2005). The implementation of PBL follows five syntax, namely: (1) orienting students to the problem; (2) organising students for learning; (3) guiding individual and group investigations; (4) developing and presenting work; and (5) analysing and evaluating the problem-solving process (Arends, 2012). The integration of UbD and PBL is shown in Table 2.

Table 2. Integration of UbD Stages and PBL Syntax

UbD Stage	Implementation with PBL
Desired Results	Determining the level of conceptual understanding of the circle equation
Assessment Evidence	Developing pre-tests, post-tests, observation sheets, and assessment rubrics
Learning Plan	Implementing the PBL syntax through contextual problem-solving activities

Once all learning activities have been completed, students are given a post-test and a feedback questionnaire. The quantitative results are then used to select participants for interviews. In the qualitative phase, observations and interviews are conducted to further explain the results obtained in the quantitative phase.

Quantitative data were analysed using descriptive and inferential statistics. Descriptive statistics were used to describe the mean, standard deviation, minimum score and maximum score. Prior to hypothesis testing, a normality test was conducted on the gain scores using the Shapiro–Wilk test, as the paired-sample t-test requires the gain scores to be normally distributed (Field, 2018). Subsequently, a paired-sample t-test was used to test for differences between pre-test and post-test scores. The hypotheses used for the paired-sample t-test were: 1) H_0: there is no significant difference between the pre-test and post-test scores; and 2) H_1: there is a significant difference between the pre-test and post-test scores. The significance level was set at $p < 0.05$.The magnitude of the treatment effect was calculated using Cohen's d for paired data, based on the mean difference divided by the standard deviation of the difference in scores. The interpretation of effect sizes followed (Cohen, 1988) criteria, namely 0.20 (small), 0.50 (moderate), and 0.80 (large).

The questionnaire data were analysed descriptively using mean values and percentages. Qualitative data obtained from observations, interviews and documentation were analysed using the interactive model proposed by (Miles et al., 2020), which comprises data condensation, data presentation and drawing conclusions. Subsequently, the results of the quantitative and qualitative analyses were integrated during the interpretation stage to gain a more comprehensive understanding of the effectiveness of implementing Problem-Based Learning within the Understanding by Design framework in the teaching of circle equations.

3. RESULTS AND DISCUSSION

3.1. Enhancement of Students' Mathematical Conceptual Understanding

The purpose of this study is to examine the effectiveness of implementing Problem-Based Learning within the Understanding by Design framework in enhancing students' conceptual understanding of circle equations. A total of 30 eleventh-grade students from SMAN 4 Malang participated in the learning activities and completed pre-tests and post-tests.

The results of the descriptive statistical analysis of the pre-test and post-test scores are presented in table 3. According to table 3, the average score for students' mathematical conceptual understanding increased from 6.06 in the pre-test to 8.21 in the post-test. Furthermore, the maximum score rose from 8.60 to 10.00.

Table 3. Description of Students' Pre-Test and Post-Test Results

Test	Number	Minimum Score	Maximum Score	Average	Variance	Standard Deviation
Pre-Test	30	1.85	8.6	6.06	2.78	1.67
Post-Test	30	2.5	10	8.21	3.95	1.99

These findings indicate a trend towards an improvement in students' conceptual understanding following Problem-Based Learning within the Understanding by Design framework. However, to determine whether this improvement is statistically significant, an inferential analysis was conducted.

Before hypothesis testing was carried out, a normality test was first conducted on the difference scores between the post-test and pre-test. The normality test was performed using the Shapiro–Wilk test as the sample size was less than 50 (Field, 2018). The hypotheses used were: 1) H_0: the difference scores are normally distributed and 2) H_1: the difference scores are not normally distributed. The results of the normality test are presented in table 4.

Table 4. Results of the Normality Test for Difference Scores (Post-Test and Pre-Test)

	Shapiro-Wilk		
	Statistic	df	Sig.
Post-Pre	.959	30	.299

Based on Table 4, a significance value of 0.299 was obtained. As the significance value is greater than 0.05 ($p > 0.05$), H_0 is accepted. Consequently, the difference in scores between the post-test and the pre-test is normally distributed, thereby satisfying the assumptions required for a paired-sample t-test (Field, 2018).

Next, a hypothesis test was conducted using a paired-sample t-test to determine whether there was a significant difference between the pre-test and post-test scores. The test results are presented in table 5.

Table 5. Results of the Paired-Sample T-Test

Score Difference	Mean Difference	t	df	Sig. (2-tailed)
Pre-test–Post-test	-2.148	-6.196	29	.000

Table 5 shows that the degrees of freedom are 29 and the significance value is 0.000 ($p < 0.05$), so H_0 is rejected and H_1 is accepted. These results indicate that there is a significant difference between the pre-test and post-test scores. To determine the magnitude of the treatment's effect, an effect size calculation was carried out using Cohen's d. The results of the analysis showed a Cohen's d value of 1.131. According to Cohen (1988), this value falls into the category of a large effect, as it is greater than 0.80.

3.2. Students' Responses to the Learning

Student responses to the implementation of PBL within the UbD framework were collected via a questionnaire comprising 10 statements covering five aspects, namely: (1) clarity of learning objectives, (2) alignment of assessment with learning objectives, (3) implementation of Problem-Based Learning, (4) use of media and worksheets, and (5) conceptual understanding and motivation to learn.

The survey results show that the majority of students responded positively to all aspects of the learning process. Regarding the clarity of learning objectives, all students agreed or strongly agreed that the learning objectives outlined at the start of the activity helped them understand the competencies they were expected to achieve. Similar findings were also found regarding students stating that the form of assessment provided was appropriate to the material and learning objectives covered.

In addition, most students also stated that the learning activities carried out boosted their confidence in determining the centre, radius and equation of a circle. The majority of students also expressed an interest in participating in similar learning models for other mathematics topics. Overall, the questionnaire results indicate that students responded positively to the implementation of PBL within the UbD framework for the topic of circle equations.

3.3. Implementation of Problem-Based Learning within Understanding by Design Framework

Based on observations revealed that the learning process was active, collaborative and student-centred. During the problem-orientation stage, students worked in small groups to analyse contextual problems relating to the equation of a circle. Most students were engaged in group discussions and participated in the process of identifying the information needed to solve the problems.

During the investigation phase, students discussed various strategies for determining the centre, radius and equation of a circle based on the information provided. The teacher acted as a facilitator, posing guiding questions and offering assistance where necessary when students encountered difficulties. Throughout the activity, students actively exchanged ideas and reviewed their group's work.

The interview results indicate that group discussion activities help students understand the concepts being studied. One student stated that group discussions made it easier for them to understand the steps involved in determining the equation of a circle, as they could discuss the matter directly with classmates who had a better understanding of the material. Another student stated that the communication of learning objectives at the start of the activity helped them to focus more closely on the learning process.

Findings from observations, interviews, and documentation indicate that the integration of PBL and UbD is capable of creating a learning environment that is active, collaborative, and oriented towards clear learning objectives.

3.4. Student Obstacles during Learning

Although the learning process generally went well, the results of observations and interviews revealed several challenges during the implementation of PBL within the UbD framework. The main challenge identified was that some group members remained less active during the discussion and problem-solving processes.

Some students tend to wait for explanations from classmates who have a better understanding of the material and do not actively express their opinions or ask questions when they encounter difficulties. This situation makes it difficult for the more active members of the group to ensure that all group members have understood the material being studied.

Nevertheless, these challenges did not hinder the learning process as a whole. Most groups were still able to complete the assigned tasks and demonstrated good engagement throughout the learning activities. These findings suggest that the implementation of PBL within the UbD framework can still be carried out effectively, although additional strategies are needed to encourage more even participation from all group members.

3.5. Discussion

The results of the study indicate that the implementation of Problem-Based Learning (PBL) within the Understanding by Design (UbD) framework has a positive effect on students' mathematical conceptual understanding of circle equations. This finding is evidenced by a significant difference between pre-test and post-test scores ($p < 0.05$). In addition to being statistically significant, the research results also show an effect size (Cohen's d) of 1.131, which falls into the 'large effect' category according to (Cohen, 1988). A large effect size indicates that the improvement in students' conceptual understanding is not only statistically significant but also has strong practical significance in the context of mathematics learning. In other words, the implementation of PBL within the UbD framework has a significant impact on students' ability to understand the relationship between geometric concepts and algebraic representations in the topic of circle equation.

This improvement in conceptual understanding can be explained by the nature of the learning process, which integrates the principles of backward design in UbD with problem-solving activities. In this learning process, students do not receive the concept of the equation of a circle directly in the form of a ready-to-use formula, but are instead invited to construct for themselves the relationship between the centre of the circle, the radius, Cartesian coordinates, and the general form of the equation of a circle through the resolution of contextual problems. This process provides students with the opportunity to build connections between concepts and mathematical representations, ensuring that the understanding formed is not merely procedural. According to Wiggins & McTighe (2005), learning designed using UbD principles is oriented towards achieving deep understanding through alignment between learning objectives, assessment, and learning experiences.

These quantitative findings are supported by the results of observations and interviews, which indicate that students were actively engaged in the learning process. The majority of students stated that group discussions helped them to understand how to determine the centre, radius and equation of a circle more easily. Through discussion, students were able to compare different problem-solving strategies, clarify misconceptions, and receive immediate feedback from peers and teachers. These findings support the constructivist view, which emphasises that knowledge is constructed through social interaction and the process of negotiating meaning during learning activities (Vygotsky, 1978).

The survey results also indicate that students responded positively to the implementation of the learning process. The majority of students stated that the learning objectives presented at the start of the activity helped them to understand the competencies they were expected to achieve. Furthermore, students felt that the forms of assessment provided were appropriate to the material and learning objectives covered. These findings indicate that the key characteristic of UbD specifically the alignment between objectives, learning activities, and assessment, has been successfully implemented. Liu (2026) explains that this alignment is one of the key factors supporting the creation of meaningful, understanding-oriented learning.

Learning success is also influenced by the use of contextual problems as a starting point for learning. In this study, students were asked to solve problems that required them to link geometric concepts and algebraic representations. These activities encouraged students to use mathematical reasoning, rather than simply memorising formulas. According to Arends (2012), the use of authentic problems in PBL can increase student engagement because they see the relevance of the material being studied to real-life situations. The results of this study are in line with the findings of Rahayu & Nugroho (2025), who reported that problem-based learning is capable of increasing students' motivation and participation in mathematics learning.

Furthermore, the use of visual media and worksheets also contributed to improving students' conceptual understanding. The questionnaire results showed that almost all students felt that visual media helped them to understand the relationship between the geometric shape of a circle and its algebraic equation. This finding is significant because one of the main difficulties students face with circle equations is linking visual representations with symbolic representations (Kpotosu et al., 2024; Niam et al., 2024). Through the visualisations and exploratory activities included in the worksheets, students can build a stronger mental representation of the concepts being studied. These results are consistent with the research by Maghfiroh & Kusumasari (2025), which showed that the use of visual media in geometry learning can improve students' spatial and conceptual understanding.

Although the research findings indicate a positive impact, several challenges were still encountered during the implementation of the learning process. Observations and interviews revealed that some students remained less active in group discussions. Students who encountered difficulties tended to remain silent and wait for explanations from other group members. This situation meant that group interactions did not always proceed optimally. This phenomenon can be explained as a form of social loafing, namely the tendency of some group members to reduce their contribution when working in a group (McQuade et al., 2019). However, the presence of passive students does not reduce the overall effectiveness of learning, as demonstrated by the increase in post-test scores and the magnitude of the effect size. Previous research has shown that this problem can be minimised through the assignment of individual responsibilities, peer evaluation, and a clear division of roles within the group (Berhanu, 2025; Lin & Chen, 2024; Putri & Syarqawi, 2024).

The findings of this study build upon previous research demonstrating the effectiveness of both PBL and UbD in mathematics education. Gürbüz & Yapıcı (2026) reported that UbD is capable of enhancing students' creative thinking skills, cognitive flexibility, and learning retention. Meanwhile, Niam et al. (2024) found that the integration of UbD and PBL can improve students' mathematical connection skills. Unlike previous studies, this study specifically demonstrates that the integration of these two approaches is also effective in improving conceptual mathematical understanding of the topic of equations of circles.

The main contribution of this study lies in the systematic integration of UbD's backward design principles and PBL syntax in the teaching of circle equations. Previous research has generally addressed PBL and UbD separately or focused on critical thinking, problem-solving, and mathematical connections (Aristanti & Fatayan, 2024; Niam et al., 2024). This study demonstrates that the integration of these two approaches is capable of creating alignment between learning objectives, assessment, and problem-solving activities, thereby supporting the development of students' conceptual understanding more effectively. These findings also address the research gap identified in the introduction regarding the limited number of studies examining the integration of PBL and UbD in the topic of circle equations.

Nevertheless, the findings of this study should be interpreted with certain limitations in mind. The study was conducted in a single class with a relatively small sample size, meaning that the generalisability of the findings remains limited. Furthermore, the use of a one-group pretest-posttest design has not fully controlled for threats to internal validity. Therefore, future research is recommended to use an experimental design involving a control group and a larger sample size to obtain stronger empirical evidence regarding the effectiveness of integrating PBL and UbD in mathematics learning.

4. CONCLUSION

This study demonstrates that the implementation of Problem-Based Learning (PBL) within the Understanding by Design (UbD) framework is effective in enhancing students' conceptual understanding of mathematics in the context of circle equations. This improvement is supported by quantitative analysis results showing a significant difference between students' initial and final abilities, and is reinforced by qualitative findings indicating that the learning process encouraged active engagement, collaborative discussion, and a deeper understanding of concepts. Furthermore, students responded positively to the clarity of the learning objectives, the alignment of assessments with learning objectives, the use of contextual problems, and the support provided by visual media and worksheets.

This study provides empirical evidence that integrating the principles of backward design in UbD with the PBL syntax can serve as an effective alternative approach to developing students' conceptual understanding of mathematics. However, some students were found to be less active in group discussions; therefore, strategies are needed to enhance participation and individual accountability in collaborative learning. Further research is recommended to involve a larger sample and a more robust experimental design to obtain more comprehensive evidence regarding the effectiveness of integrating PBL and UbD in mathematics learning

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