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# COLLECTIVE ARGUMENTATION AND PARTICIPATION IN SOLVING GEOMETRY PROBLEMS IN THE MATHEMATICS CLASSROOM

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**ABSTRACT**

Collective argumentation is a process in learning that can be used to train communication skills, collaboration, and understanding of mathematical concepts. In this process, both teachers and students play an active role, which is called participation. This study aims to describe the structure of collective argumentation and teacher and students participation in solving geometry problems in the classroom. This research method is a qualitative case study. The subjects in this study were a mathematics teacher with 11 years of teaching experience at the junior high school level and six 9th-grade students who had an interest in mathematics from two different classes. The structure of collective argumentation shows that this learning focuses on students while the teacher acts as a facilitator. It can be seen from the more significant number of actions taken by students than teachers. In terms of participation, teachers more often act as ghostee, while students participate more as spokesman. Overall, this study reveals the structure of argumentation in solving geometry problems at each stage of Polya. Questions and explanations given by the teacher influence students' collective argumentation. A teacher must have questioning and communication skills so that students can actively participate in learning in the classroom.



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## 1. INTRODUCTION

Problem-solving skills are still the main focus in mathematics classes. In mathematics classes, problem-solving learning tends to emphasize the correct final answer (Noviyanti & Suryadi, 2019). Teachers tend not to provide space for class discussion. In

fact, some students have difficulty expressing what the problem is, what they need, and how to solve it (Uzun, 2024). Without guidance or intervention from the teacher, students who have difficulty understanding the material tend to remain in a state of confusion. Likewise, for students who already have sufficient understanding, without intervention in the form of feedback from the teacher, even though students have the correct solution, they will feel doubtful about the solution they already have. Learning that should provide a correct and robust understanding of concepts becomes something that is doubtful or maintains wrong concepts. Learning like this is not expected to happen in the classroom.

argumentation refers to the process by which two or more individuals interact to construct or attempt to construct a claim (Krummheuer, 1995). Collective argumentation has been studied in education, and it has been found that this process can support mathematics learning and is considered effective in instilling conceptual understanding (Castro, 2023; Civil & Hunter, 2015; Conner & Singletary, 2021; Krummheuer, 1995, 2007; Ness & Maher, 2023; Nickel, 2019). Argumentation skills are 21st-century skills that students need to have because they involve problem-solving, making statements, making decisions supported by data and evidence, and forming ideas (Bahri et al., 2021; Campbell et al., 2020; Putri et al., 2024).

Collective argumentation can be an effective way to help students develop critical thinking, reasoning, and collaboration skills as a result of better understanding through structured and meaningful interactions (Cardetti & Lemay, 2018; Demiray, 2023). Argumentation should be a learning goal. Teachers should create a fair classroom environment that supports the exchange of ideas and debates, where students have many opportunities to justify their mathematical thinking and develop rigor in solving mathematical problems (Francisco, 2022; Krummheuer, 2007). In general, argumentation can be defined into two main perspectives: argumentation as a social negotiation or debate process (social perspective) and argumentation as a cognitive process to provide assurance and supporting evidence for claims (cognitive perspective) (Zhou et al., 2021). In this study, argumentation is defined as a thinking process that occurs in mathematics classes, where students exchange ideas to explain and justify their answers in solving mathematical problems. The goal is for students to be able to understand mathematical concepts better through discussions and exchanges of ideas carried out together.

Brown (2017) commented that collective argumentation "has the potential to create a communicative space in the classroom where students have regular opportunities to 'represent', 'compare', 'explain', 'justify', 'agree' and 'validate' their ideas" (p. 186). In the process of collective argumentation, students can contribute through active participation in argumentation activities, build arguments, and be responsive to the contributions of their peers (Wilkie & Ayalon, 2023).

Research on argumentation has significantly contributed to the development of an understanding of how students construct and present arguments in educational contexts, especially in mathematics subjects. Kartika & Budiarto (2022) conducted a study to investigate the quality of students' argumentation in solving mathematical problems. A total of 41 junior high school students in Indonesia solved an argumentative task. The students' responses were then analyzed to determine the quality of their arguments. The results of the study revealed that more than half of the students misunderstood the given mathematical problems. Furthermore, Uzun (2024) conducted a case study in two mathematics classes to describe the development of the movement. Arguments in both classes and assess what kind of argumentative dialogue is conducted by the teacher. The results of the study showed the inadequacy of collective discussion and quality discussion. Uzun's (2024) study showed the formation of arguments naturally in both classes. This study assessed teachers and students, while Kartika's (2022) study shows the quality of students' arguments in solving

mathematical problems. This study was conducted by giving mathematical assignments to 41 students. The students were asked to write their responses on the sheet provided.

The author is interested in doing something similar to Uzun's (2024) and Kartika & Budiarto (2022) research. However, what distinguishes this study from the two previous studies is its focus on describing the arguments and participation that occur in the classroom when solving mathematical problems. It provides instructions to teachers to direct learning activities according to Polya's stages, namely understanding the problem, making a plan, implementing the plan, and reviewing the results that have been obtained. In addition, researchers also saw the participation of students and teachers in collective argumentation activities in mathematics classes.

The argumentation activity in this study is connected to the dialogue process that occurs through the interaction between ideas that strengthen or different ideas so as to form a collective argument that can shape students' understanding in solving geometry problems. Therefore, this study contributes to the literature in the field of mathematics class argumentation. The research questions are as follows: (1) How is the structure of collective argumentation in solving geometry problems in mathematics class? (2) how do teachers and students participate in argumentative activities to solve geometry problems in mathematics class?

## 2. METHOD

A qualitative case study was conducted in this study. Six students who had a high interest in mathematics were selected from two different classes by asking the students' mathematics teachers. The six students were grade 9 students who had received materials on plane figures and the Pythagorean theorem. The teachers selected in this study had 11 years of junior high school teaching experience. The instrument in the study was in the form of questions adopted from Mutammam et al. (2023). The two focuses of this study are describing the formation of collective argumentation in the classroom and participation in collective argumentation.

Data analysis consists of two parts: the initial part involves coding the data, and the next part categorizes students' participation in argumentative dialogue. Collective argumentation in the classroom was recorded on video, transcribed, and analyzed. To analyze the structure of collective argumentation, the study adapted the research coding scheme (Ayalon & Even, 2016). The coding of collective argumentation components in the classroom can be seen in Table 1.

**Table 1.** The coding of collective argumentation components in the classroom

| Component | Description | Example of class |
|---|---|---|
| Claim | The Speaker's statements (in the form of assumptions, generalizations, conclusions, ideas, and facts) | What is known is a polygon image, sir? |
| Request for a claim | Requests to submit conjectures, generalizations, conclusions, ideas, and facts | What is known about this problem? |
| Justification | Support, explanation, or evidence provided for a claim | It is known that the polygon image has the same distance between adjacent points of 1 unit. |

| Component | Description | Example of class |
|---|---|---|
| Request for justification (RJ) | The urge to justify claims | It is the same |
| Elaboration of justification | Additional information that further supports the claim | It is incomplete, sir. What is known about question number 1 is that the distance between adjacent points vertically and horizontally is one unit. |
| Challenge for evaluating | Encouragement to evaluate students' statements and ideas | Wait. What does vertical mean? |
| Agreement | Statements expressing support for a previously submitted statement | This is close. It is straight along the line. |
| Opposition | An utterance that expresses disagreement with a particular statement made previously. | Build the bottom instead of a trapezium. It is a trapezium, not a square |
| Repetition | Repeating previous words | G: Agreed?<br>S1: No, sir. Those are two.<br>S2: Yes, those two |
| Concession (Co) | A statement of agreement with a discussion that the same person previously opposed. | That means I was wrong, sir. I counted many points. It turns out the jump. |
| Approving | Statement of acceptance of previously submitted claims or justifications | G: Agree with Tio?<br>S1: Yes, sir, I agree.<br>S1: agree |

Meanwhile, Krummheur's (2007) research was adapted to analyze student and teacher participation(2007)(2007). Table 2 is categories of students and teachers in collective argumentation in the classroom.

**Table 2.** Categories of students and teachers in collective argumentation in the classroom

| Category | Description | Example of class |
|---|---|---|
| Author | A person takes full responsibility for the content and formulation of their statements, and they convey their ideas in their own words. | It is not vertical and horizontal. |
| Relayer | A person who is not responsible for the content or formulation of his statements and only restates other people's statements without changes. | Yes, It is not vertical and horizontal |
| Ghostee | A person who uses another person's words to convey a new idea and takes the formulation of a previous statement but gives it a different meaning. | |
| Spokesman | Someone who takes an idea from a previous statement and puts it into their own words | From the picture, it moves up and down, sir. |

## 3. RESULTS AND DISCUSSION

### 3.1. Results

Argumentation process in solving problems in mathematics class was carried out for 120 minutes. Teachers were asked to teach the steps of solving geometry problems centered on students. In this activity, students were directed to argue as much as possible about the steps of solving geometry problems given by the teacher.

*Collective argumentation structure*

Stage 1: Understanding the problem

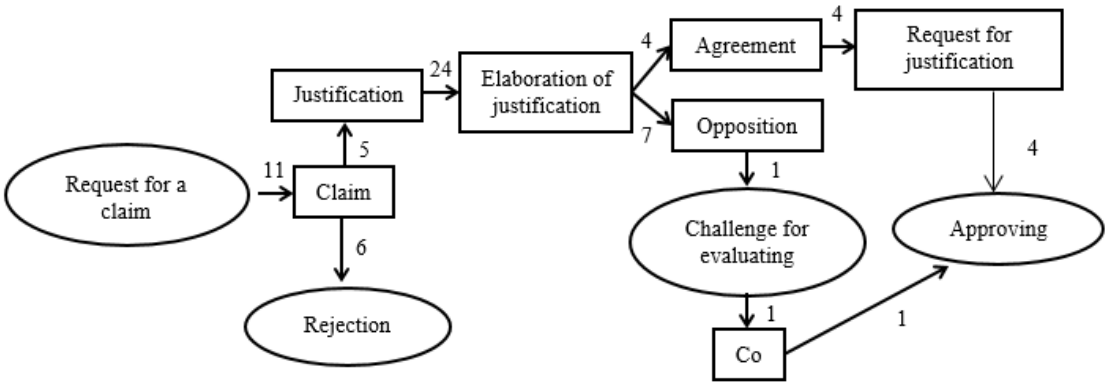



**Figure 1.** Collective argumentation at stage 1: Understanding the problem

Figure 1 shows the argumentation structure of all discussions in a mathematics class in solving problems at the stage of understanding the problem. The circular shape symbolizes the teacher's actions, while the rectangular shape in the picture shows the student's actions. The arrows indicate the direction of the argumentation movement, and the numbers on the arrows indicate the number of actions. Figure 1 illustrates that the teacher made claims 11 times. Six times, the student's claims were rejected because they were not appropriate, and 5 times, the claims were accepted. Claims that the teacher rejected were not evaluated. The teacher gave students the opportunity to have a dialogue. Students responded to appropriate claims by providing evidence, and other students provided additional information to strengthen the evidence of the claim 5 times. The results of the additional information to strengthen the evidence turned out to be not all true. There was one additional piece of information from the student that was not appropriate. The student tried to explain but was constantly opposed by other students. The teacher was present to evaluate statements from students' ideas that were not appropriate by asking claims or questions to all students 1 time. However, some answers to the questions were not appropriate and were rejected 6 times. Students gave re-explanations and were appropriate to clarify the claims. At the end of the discussion, the teacher accepted all claims given by students. It shows that all students have understood the problem given.

Stage 2: Plan problem resolution

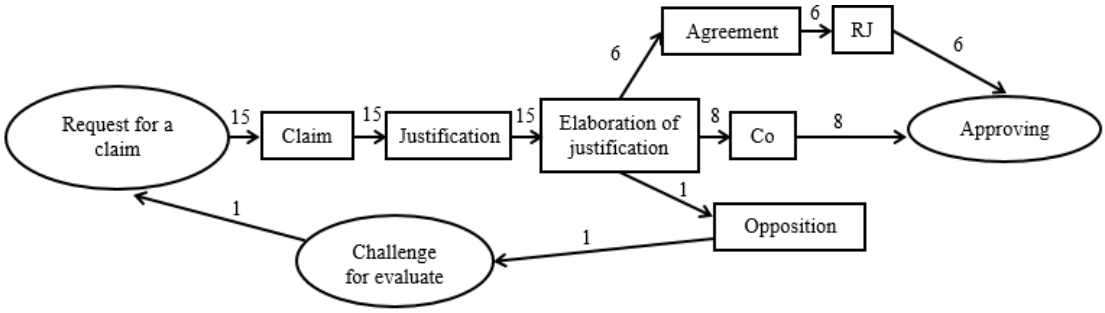



**Figure 2.** Collective argumentation at stage 2: Planning problem solving

Figure 2 explains that the teacher submitted claims 15 times, with the initial claim 7 times, and all claims were accepted. The teacher gave students the opportunity to have a dialogue. Collective argumentation at this stage explains that there are two cycles. In the initial cycle, students responded to appropriate claims by providing evidence, and other students provided additional information to strengthen the evidence of the claim 15 times. The results of additional information to strengthen the evidence turned out to be not all true. There was one additional piece of information from a student that was not appropriate. The student tried to explain but was constantly opposed by other students 6 times. Students whose arguments were accepted re-explained their ideas. In cycle 2, the teacher was present to evaluate statements from students' ideas that were not appropriate by submitting claims or questions to all students 1 time, and all students actively provided support for claims that were approved 6 times. With the evaluation from the teacher and support from students' statements, students with wrong arguments agreed to the arguments that were previously opposed. At the end of the discussion, the teacher accepted the claims given by the students. It shows that all students have planned to solve the problem correctly.

Stages 3 and 4: Carrying out the plan and looking back the solution

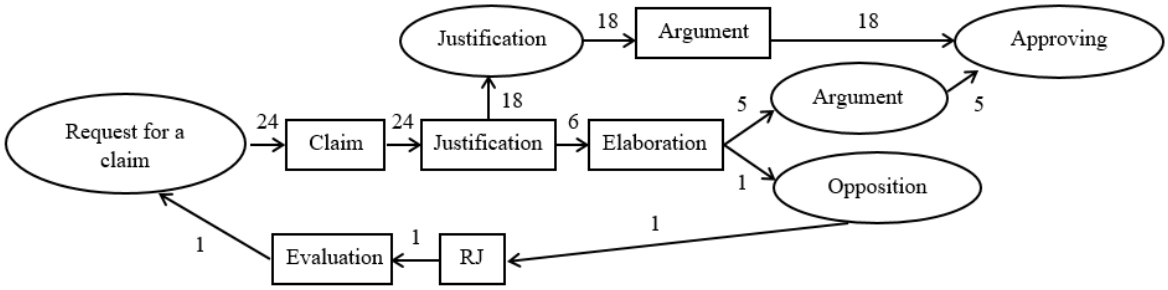



**Figure 3.** Collective argumentation at stages 3 and 4: Carrying out the plan and looking back the solution

There are two cycles of collective argumentation in Figure 3. Cycle 1 explains that the teacher submitted claims 6 times, and all claims were accepted. Students responded to the appropriate claims by providing evidence, and other students provided additional information to strengthen the evidence of the claims 6 times. The results of the additional information to strengthen the evidence turned out to be not all correct. Five additional pieces of information were accepted and approved by the teacher. In cycle 2, it started with one additional piece of information that the teacher rejected. Students took the action of "requesting for justification." The teacher acted to evaluate one student's argument that was rejected. The teacher submitted 18 claims, and students provided or answered the claims and explained the existing claims. The teacher explained the claims by emphasizing the correct explanation of the claims from the students as many times as possible. Students accepted the

explanation of the claims and were accepted by the teacher. It shows that all students have carried out the stages of solving the problem and checking the answers correctly.

*Student and teacher participation*

The subjects in this study were six selected students. The six students were students with different levels of ability. All students actively participated in the discussion and wrote down the solution to the problem on the answer sheet. The participation of students and teachers in collective argumentation to solve problems in mathematics class, namely:

- Student 1 tends to be an author at all stages. He always gives responsible responses and uses his own words. Student 1 often responds quickly and accurately. The following transcript excerpt illustrates that student 1 tends to be an author.

| | | |
|---|---|---|
| Teacher | : | *Agree with R's opinion* |
| Student 1 | : | *It is incomplete, sir. What is known about question number 1 is that the distance between adjacent points vertically and horizontally is one unit.* |
| Student 2 | : | *It is the same* |
| Student 1 | : | *Different. What is known in the question is that the distance between two points is one unit only vertically and horizontally.* |
| Student 4 | : | *Pack. I want to show what is known. Is it allowed?* |
| Teacher | : | *Yes, please* |
| Student 4 | : | *So what is known is that this polygon shape (pointing to the image on the projector display) is the same as the distance from this point to this point 2 units (students point to the first point with the second point, which is adjacent)* |
| Student 1 | : | *Hey, why that one?* |

  Student 1 tends to be in the author category because, in several conversations, Student 1 always responds to other students' and teachers' answers using his ideas and words. When the teacher asks for opinions on the results of other students, with his ideas and words, Student 1 states his opinion and adds information that he thinks is appropriate. Student 1 sticks to his position in explaining the answer. Without being asked to ask, Student 1 also immediately opposes other students' answers that are considered inappropriate from the question. Student 1 tends to be an author who expresses answers by thinking first so that the explanations given are primarily appropriate.
- Student 2 tends to be in the author, relayer, and spokesman categories at all stages of problem-solving. Here is an excerpt from the transcript

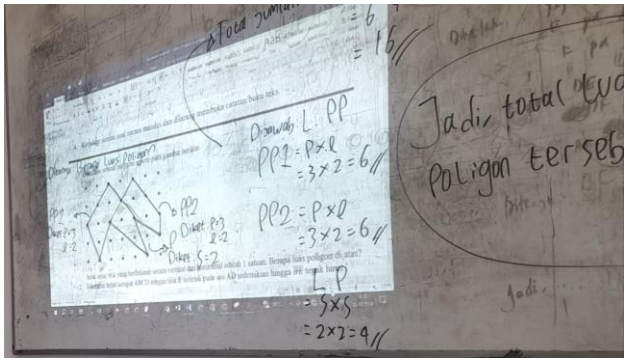
| | | | |
|---|---|---|---|
| Teacher | : | *Wait. What does vertical mean?* | |
| Student 1 | : | *The vertical one is the one that stands, sir.* | |
| Student 2 | : | *Yes, vertical is a line that stands like this (for example, with his hands)* | Spokesman |
| Teacher | : | *What about this picture?* | |
| Student 1 | : | *From the picture, it moves up and down, sir.* | |
| Student 2 | : | *Yes, it moves up and down* | Relayer |
| Student 1 | : | *Yes, straight up and down cannot be slanted* | |
| Teacher | : | *What is horizontal?* | |
| Student 2 | : | *The sleeping one* | Author |

| | | | |
|---|---|---|---|
| Student 1 | : | *That line is horizontal* | |
| Student 3 | : | *Yes, like this (demonstrated with his hands), his hands are horizontal.* | |
| Teacher | : | *What do you think about this picture?* | |
| Student 1 | : | *That is right and left, sir.* | |
| Student 2 | : | *Forward to the right, backward to the left* | Spokesman |

In the transcript excerpt above, it is described that student 2 is included in the spokesperson category because student 2 uses the answer from student 1. However, student 2 expressed it in a different sentence, namely indicating a vertical line by moving his hands. In another conversation, student 2 also uses student 1's answer but expresses it in a different sentence, namely, illustrating moving forward to the right and moving backward to the left. Student 2 is also included in the relayer category because student 2 answers the teacher's question with the same idea and meaning as student 1. Student 2 is also included in the author category because student 2 answers the teacher's question using his sentences and ideas.

- Student 3 tends to be in all categories, namely author, relayer, spokesman, and ghostee, at all stages of solving the problem. Here is an excerpt from the transcript

Student 4




*There are three shapes formed. It is a rectangle; the area of rectangle 1 is length times width. The length is 3 (counting the number of points), and the width is 2 (counting the number of points), so 6. It is the second rectangle. The length is 3; the width is 2, so 6. Then, this is a square. The area of a square s x s, or this is length and width. The first side is 2; the second side is 2, so 4. The area of the polygon is the total area. 6 plus 6 plus 4 equals 16. So, the total area of the polygon image is 16 units*

| | | |
|---|---|---|
| Student 3 | *Rectangle* | Ghostee |
| … | | |
| Student 1 | *It is also not horizontal or vertical.* | |
| Student 3 | *Yes, the horizontal and vertical are one unit.* | Spokesman |
| Student 4 | *This one is straight horizontal (while pointing to the length of the first rectangle), this one is vertical (while pointing to the width of the first rectangle)* | |
| Student 1 | *It is not vertical and horizontal.* | |
| Student 3 | *Yes, It is not vertical and horizontal* | Relayer |
| … | | |
| Teacher | *What is horizontal?* | |
| Student 2 | *The sleeping one* | |

| | | |
|---|---|---|
| Student 1 | *That line is horizontal.* | |
| Student 3 | *Yes, like this (demonstrates with his hands), his hands are horizontal* | Spokesman |

In the transcript excerpt above, it is described that Student 3 is included in the spokesperson category because Student 3 uses the answer from Student 1. However, Student 3 expresses it in a different sentence by adding appropriate information. In another conversation, student 3 also uses student 1's answer but expresses it with a different sentence, namely gesturing horizontally by moving his hands to sleep. Student 3 is also included in the relayer category because student 3 answers the teacher's question with the same idea and meaning as student 1. Student 3 is also included in the ghostee category because Student 3 expresses disagreement with Student 4's answer by saying the same words as Student 4.

| | | | |
|---|---|---|---|
| Teacher | : | *What do you think? Agree with this picture.* | |
| Student 6 | : | *Yes, sir, I agree* | |
| Student 3 | : | *I am different, sir. I have nine shapes, sir.* | Author |

The transcript excerpt above also shows that student 3 is included in the author category. He explains the answer with his own ideas and sentences.

- Student 4 tends to be in all categories, namely the author, at all stages of solving the problem. Here is an excerpt from the transcript

| | | | |
|---|---|---|---|
| Teacher | : | *What shape is this building?* | |
| Student 3 | : | *Cut the rice cake* | |
| Student 4 | : | *Rectangle* | Author |

Student 4 tends to be in the author category because, in several conversations, student four always responds to other students' and teachers' answers by using their ideas and words even though the ideas are not quite right. Student 4 tends to be an author who expresses answers without thinking much.

- Student 5 tends to be in all categories, namely relayer at all stages of solving problems. Here is an excerpt from the transcript

| | | | |
|---|---|---|---|
| Teacher | : | *What is being asked about this?* | |
| Student 1 | : | *Area of polygon, sir* | |
| Student 5 | : | *Yes, the area of the polygon* | Relayer |

Student 5 tends to be in the relayer category because student four answers the teacher's questions with the same ideas and meanings as student 1. In argumentation activities, student 5 tends to be passive in answering. However, every time he answers, he always thinks about his answer first and provides a logical explanation, even though, on several occasions, he answers incorrectly.

- Student 6 tends to be in all categories, namely author and spokesperson, at all stages of solving the problem. Here is an excerpt from the transcript

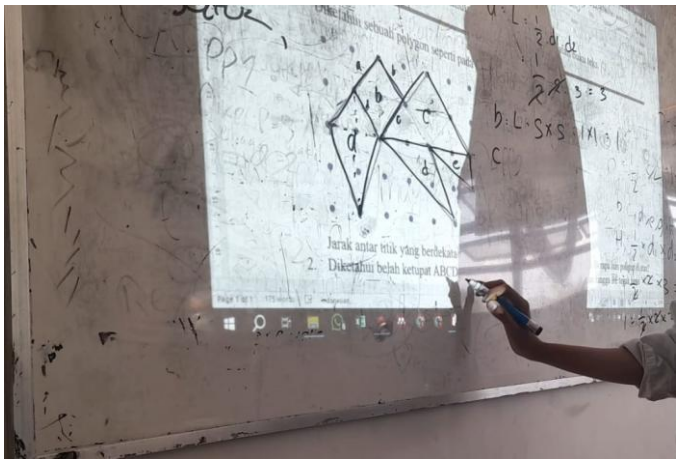
| | | |
|---|---|---|
| Teacher | : | *Let us save it first. According to Wulan, the area of the nine shapes she formed from this polygon is 12 units. Do you have any other answers?* |

| | | | |
|---|---|---|---|
| Student 6 | : | *I sir. I made five shapes* | Author |
| … | | | |
| Teacher | : | *Yes, please* | |
| Student 4 | : | *So what is known is that this polygon shape (pointing to the image on the projector display) is the same as the distance from this point to this point 2 units (students point to the first point with the second point, which is adjacent)* | |
| Student 1 | : | *Hey, why that one?* | |
| Student 2 | : | *Not that* | |
| Student 4 | : | *Yes, that is true; it is known.* | |
| Student 3 | : | *Not that* | |
| Student 6 | : | *Yeah, that is not it. It should be 1 unit.* | Spokesman |

Student 6 tends to be in the author category because he answers the teacher's questions with his ideas. Student 6 is also included in the spokesperson category. Student 6 responds to the answer from the explanation of Student 4 by agreeing with the same answer as Student 1, student 2, and Student 3 and adding information related to the question. In argumentation activities, student 6 tends to be passive in answering, but every time he answers, he always thinks about his answer first and provides a logical explanation.

- Teachers are in all categories, namely author and ghostee, at all stages of solving problems. Here is an excerpt from the transcript

Student 6 :



*I made five shapes. Shape A is a kite. The area of a kite is half a kite multiplied by diagonal one multiplied by diagonal 2. Diagonal 1 is two units, and diagonal 2 is three units. The area of kite 3. Shape b is a square. The area of a square is side times side. The side is one, so one time, one is equal to one.*

| | | | |
|---|---|---|---|
| Teacher | : | *Agree with build b* | Author |
| Student 2 | : | *Agree* | |
| Student 1 | : | *No* | |
| Teacher | : | *Is it true that this is a square with a side length of one?* | Author |
| Student 1 | : | *I disagree, sir. Earlier, we agreed that it should not be slanted, but the sides must be the same.* | |
| Teacher | : | *The sides must be the same* | Ghostee |
| Student 1 | : | *Yes* | |

Teacher tend to be author and sometimes become ghostee. He become ghostee by re-emphasizing what students say. It suggests that students are asked to rethink their answers because they are wrong or incomplete. In addition, in some classroom conversations, teachers become spokespersons to emphasize that this answer is correct so that it gets the attention of all students.

From six students, the most active student who provides logical and correct explanations is student 1. In the transcript of argumentation activities, student 1 responded to questions from the teacher or other students 41 times and refused by explaining if there was an explanation that was not appropriate. Students 5 and 6 tended to be passive in discussion activities, but students 5 and 6 always tried to respond to questions and explain answers logically and tended to be appropriate. If they could not answer, they would be silent and think about the answer. Student 4 became an active author participant, but his opinions and ideas were often inappropriate.

### 3.2. Discussion

In the stage of understanding the problem, shows that the learning carried out by the teacher is student-centered. It can be seen from the argumentation scheme that student actions are more than teacher actions. As stated by Krummher (2015), the characteristics of teachers in teaching are usually to allow students to discuss and argue and to participate when something goes wrong. However, the teacher is not very good at asking questions that refer to the justification of claims. From the argumentation scheme of 11 claims submitted by the teacher, five claims can create discussions, and 6 claims are rejected without any other action from the teacher. In fact, the questioning strategy is a vital tool to use in an argumentation learning environment (Berland & McNeill, 2010). Asking "how" and "why" questions is essential for creating class discussions (McCrone, 2005).

The teacher's denial of the claim suggests that the student lacks knowledge or understanding of the issue. The student does not understand some terms that the teacher considers familiar. In the argumentation scheme, it is explained that students play an active role in argumentation because the claims made by students are always followed by the actions of their students and other students. In addition, the argumentation scheme shows that teachers are present when students experience difficulties or confusion in the justification made being rejected by other students. In the classroom, the role of teachers is crucial in creating an argumentative learning environment (Jiménez-aleixandre, 2007; McCrone, 2005).

At the stage of understanding the problem, students accept all claims made by the teacher. The teacher allows students to have a dialogue. The results of additional information to strengthen the evidence turned out to be not all true. One additional piece of information from the student needs to be corrected. The student tried to explain but was continuously opposed by other students. The student whose argument was opposed explained his idea again because he believed in the truth of the claim he had put forward. As mentioned earlier, teachers are usually present when something is wrong (Krummheuer, 2015). In this case, student discussion does not make a student with a different opinion accept that the claim is wrong. Teachers can support students by providing argument components, using various questions and other supporting actions, and adapting their contributions and support in different situations (Menke et al., 2024). In this case, the teacher is present to evaluate statements from students' ideas that could be more right by submitting claims or questions to all students. After that, all students actively provide support for the approved claim. With the evaluation from the teacher and support from the student's statement, the student whose

argument was wrong agreed to the previously opposed argument. At the end of the discussion, the teacher accepted the claim given by the student. It shows that all students have planned to solve the problem correctly.

Each person who participates in a discussion may be in more than one category, and the statement given by a participation in one sentence also allows the same thing. Determining the appropriate participation category requires reading the previous and following dialogues.

This study has limitations. Namely, it is only limited to students and teachers who have been determined. The analysis was carried out only on selected students, one teacher, and one learning session. It is considered insufficient. More observations in the classroom will deepen our understanding of teachers' tools and strategies for developing quality and collective argumentation in the classroom. Thus, the findings in this study cannot direct researchers to generalize the state of mathematics classes in all schools, which always conduct active collective class discussions and quality argumentation. The findings of this study can only help teachers reflect on the results and find ways to improve the situation.

The results of this study have implications for practice and research. First, the importance of teacher questioning skills in argumentation. Questions delivered by teachers in argumentation activities affect class conditions and student participation. If teachers deliver simple questions with passive participation, it will make arguments in understanding mathematical concepts only occur in one direction, namely centered on the teacher. Teachers must be able to provide appropriate procedural questions and build student arguments in one class. Second, in learning, teachers should be facilitators in argumentation classes. Teachers must be able to resist the urge to make claims if there are opinions from students that go off track. Teachers can divert by giving procedural questions. Third, collective argumentation is essential to apply in class because argumentation activities can increase teacher knowledge about student understanding of a material and, at the same time, a means for teachers to straighten out concepts that are not yet appropriate. Fourth, collective argumentation is an effort to improve student understanding of mathematical concepts through arguments in class. Fifth, systematically train students in solving mathematical problems with the Polya stage.

## 4. CONCLUSION

The overall structure of collective argumentation in solving geometry problems shows student-centred learning. At the stage of understanding the problem, collective argumentation is dominated by student actions while the teacher tries to be a facilitator. Students try to build a shared understanding with the teacher as a means to correct their inaccurate understanding. When there are students who have an inaccurate understanding, the teacher and other students try to provide evaluation and justification actions until the student accepts the answer that was previously opposed.

At the planning stage, students make many claims, and the teacher does not reject them. All of these claims are continued with several student actions. The teacher is present to evaluate students' answers by submitting claims in the form of questions to all students. The questions strengthen additional appropriate information. The teacher tries to evaluate by positioning himself as a facilitator.

The third and fourth stages are problem-solving and rechecking answers. At this stage, the teacher takes more action. When students elaborate on the justification, the teacher justifies and rejects the action. In the student's justification action against the claim, the teacher justifies it, and the students agree with the justification. The justification made by the teacher is to check students' understanding through related questions.

Six students and one teacher participated in collective argumentation. Teacher is dominant in the author category and several times as ghostee. Teachers become ghosties when students' answers are not correct. Overall, students tend to be authors and are followed as relayers. Two students tend to be passive compared to the others but are active in solving the questions on the answer sheet.

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