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COMMOGNITIVE ANALYSIS OF STUDENTS' WORK IN SOLVING PROBABILITY PROBLEMS

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ABSTRACT

High school mathematics has a gap with university-level mathematics. At the university level, students are expected to have strong oral and written communication skills, as well as critical and creative thinking abilities, which are reflected in their problem-solving work. Students' work can be analyzed using the commognitive lens, encompassing the components of word use, visual mediators, narratives, and routines. This study aims to describe students' cognitive abilities in solving probability problems. This research is a qualitative study with the researcher as the primary instrument. A total of 10 students from a private university in Indonesia were given one essay problem to complete within 30 minutes. Their work was analyzed using commognitive analysis to describe their problem-solving abilities. The next step involved grouping based on their work, resulting in two students selected as research subjects, who then participated in semi-structured interviews to confirm their answers in their written work. The results showed that the students' ability to solve probability problems showed proficiency. The students could translate problem statements into easily understandable sentences (word use). They were also able to illustrate the information given in the problem with correct visuals (visual mediators). Based on their prior knowledge (routines), the students correctly used formulas to determine the probability of an event and drew conclusions (narratives). Further research can be developed to reduce differences of opinion that trigger cognitive conflict.

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1. INTRODUCTION

Math in high school is not the same as math in college or university (Emanuel & Meilantifa, 2022; Putut, Emanuel, Nusantara, & Rahardi, 2024; Putut, Emanuel, Nusantara, Rahman, & Rahardi, 2023). Students who study mathematics in college must possess critical thinking, creative thinking, and good oral and written communication skills (Kim, dkk., 2019; Thoma & Nardi, 2017, 2018). Thinking and communicating are one and the same (Ioannou, 2016, 2017; Presmeg, 2016). Commognitive is a novel concept that arises from the union of communication and cognition (Sfard, 2008, 2020). The four components of commognitive processes are routines (N), narratives (N), visual mediators (VM), and word use (WU) (Lu, Zhang, & Stephens, 2019; Tabach & Nachlieli, 2016)*).*

The narratives that arise in students' work exhibit these four elements. The usage of words or sentences in both daily life and student work is known as word use (WU) (Mpofu & Pournara, 2018; Viirman & Nardi, 2019a). WU can be expressed orally or in writing as a word, sign, numeral symbol, symbol, or mathematical statement (Zayyadi, Nusantara, Subanji, Hidayanto, & Sulandra, 2019).

The visual mediators employed in mathematical work results are called Visual Mediators (VM) (Pratiwi, Nusantara, Susiswo, & Muksar, 2020). VM can be actual things, variables, tables, graphs, or diagrams. The justifications or arguments that students utilize in their work are called narratives (Lu, Tao, Xu, & Stephens, 2020).

Mathematical rules or theorems can be narratives. Conversely, routines are recurring patterns in problem-solving, such as methods for resolving a mathematical issue (Halim, Nurhidayati, Zayyadi, Lanya, & Hasanah, 2020). Student work can be examined using these four elements.

The process of applying a commognitive framework to examine student work in problem-solving is known as cognitive analysis. Word use, visual mediators, narratives, and routines are the four commognitive components that make up the commognitive framework. These four elements make up cognitive analysis, which has been routinely utilized to evaluate student work (Ioannou, 2018; Kim,dkk., 2019b; Viirman & Nardi, 2019b). Building a formal knowledge of concepts is frequently hampered by the transition from school mathematics to university mathematics (Kim et al., 2019b). Researchers were compelled to investigate this phenomena because of concerns about potential flaws in student work. The purpose of this study is to characterize how first-year students solve probability issues in a commognitive manner. There have been numerous studies on first-year students, but none have detailed the students' cognitive skills in this subject, and even fewer have looked at how commognitive analysis is applied to the subject of probability, particularly in first-year students. This content is covered in the course on discrete mathematics.

2. METHOD

The primary instrument in this qualitative study is the researcher (Cresswel, 2013). Commognitive analysis serves as the primary analytical framework in this descriptive qualitative study(Sfard, 2008, 2020). With the use of student workbooks, a semi-structured interview guide, and a recording device for audio-visual documentation, the researcher herself served as the primary instrument in this study. Four components were identified in order to examine the data using a cognitive analytic approach: routines (R), narratives (N), visual mediators (VM), and word use (WU). The researcher came to conclusions about the research problem based on the findings of the data analysis (Kelly, 2014). Based on their availability for interviews and active involvement in courses, ten current Mathematics Education majors from a private university in Indonesia were purposefully chosen. By

comparing student work, interview transcripts, and firsthand observation through visual documentation, triangulation was carried out.

In the first step, participants were given 30 minutes to complete individual probability questions. Grouping responses according to response patterns was the second step. Interviews with representatives of each group were conducted in the third stage. **Figure 1** displays the questions that were asked.

> The following findings were obtained from a survey of 35 village residents.
> Tea is preferred by 18 residents, coffee by 17, milk by 14, tea and coffee by 8, tea and coffee by 7, coffee and milk by 5, and all three by 3. If two inhabitants are chosen at random, confirm that the likelihood that they both favor neither milk nor neither is $\frac{117}{119}$.

Figure 1. Practice Questions for the Probability Content

The students' work was then grouped according to the types of answers and analyzed. Next, from each group, one subject was selected to be interviewed using a semi-structured interview based on their work. This was done to gain deeper insight into the students' commognitive processes. The reason for choosing the semi-structured interview model was to help ensure that the research subjects did not feel they were directly involved in a formal research study.

3. RESULTS AND DISCUSSION

3.1. Results

Following thirty minutes of working on contextual maths problems, students' work was chosen based on how accurately they answered the questions. Based on their work, one student was chosen as a research subject from each of the two groups. Following that, each subject's results were examined using a commognitive framework, which produced the following findings.

Figure 2 shows that, based on prior experience (R-1), S1 is able to write the information provided in the inquiry using common words or sentences (WU-1). Based on the repeated pattern that has been completed (R-1), S1 may also accurately depict the known circumstances in the question in a Venn diagram (VM-2). Furthermore, while correctly answering the question, S1 is able to use symbols (VM-3). S1 applies the same pattern based on prior knowledge (R-1) while answering the question. S1 calculates the likelihood of appropriately taking two at once using a formula (N-2). Last but not least, S1 correctly calculates the probability formula for an event and its complement (N-1).

It can be inferred from S1's work that S1 is capable of accurately resolving the issue. The following commognitive review was used to assess the clarification of S1's responses from semi-structured interviews between the researcher (P) and the first subject (S1).

P : *Please provide an explanation of your work findings (referencing your S1 work results)*

S1 : *All OK, sir. I started by writing down what I knew about the issue after reading it and comprehending it. What is the number of individuals present? Tea, coffee, milk, tea and coffee, tea and milk, coffee and milk, how many people like these? To make it easier to understand, I*

then illustrated it using a Venn diagram. I followed the procedures I had previously learnt to accomplish this.

P : What is next?

S1 : I then finished the Venn diagram. To make it simpler to see what was lacking in the picture, I then specified a few variables. I completed this step to resolve the prior issue, and it made sense to me. The Venn diagram was finished at last.

P : So, how is the probability determined? Describe!

S1 : All right, sir. I'll remember how to calculate the likelihood of taking two at a time because there are five people who only want milk. I'll employ a blend based on what I've learned thus far. I'll get the probability value from this.

P : Then how?

S1 : Okay, sir. After reading the question again, I utilized the formula that says that the probability of an occurrence plus its complement equals one to figure out the answer. I therefore solved the issue and came to a decision using that rule.

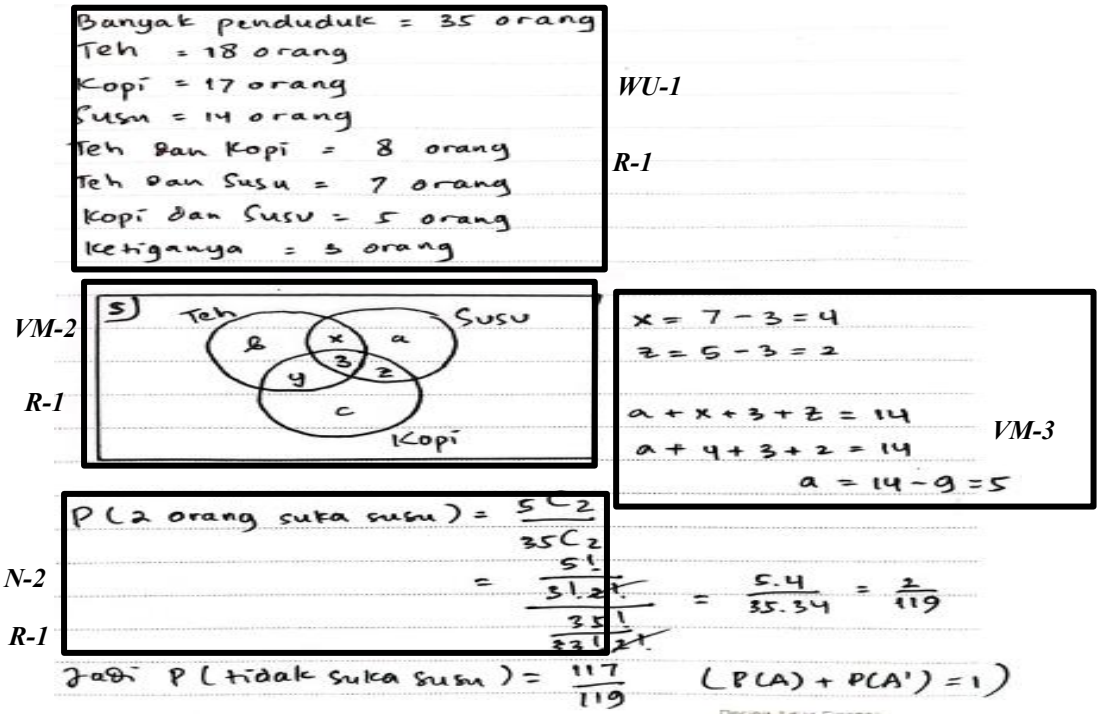

Figure 2. Subject 1 Work Results (S1)

Figure 3 shows that, according to prior experience (R-1), S2 is able to employ everyday words correctly (WU-1). Based on the previously learned steps (R-1), S2 can accurately depict them in a Venn diagram (VM-2). The formula can also be used by S2 to calculate the likelihood of properly taking two at once (N-2). Lastly, S2 can accurately make inferences using the probability formula for an event and its complement (N-1).

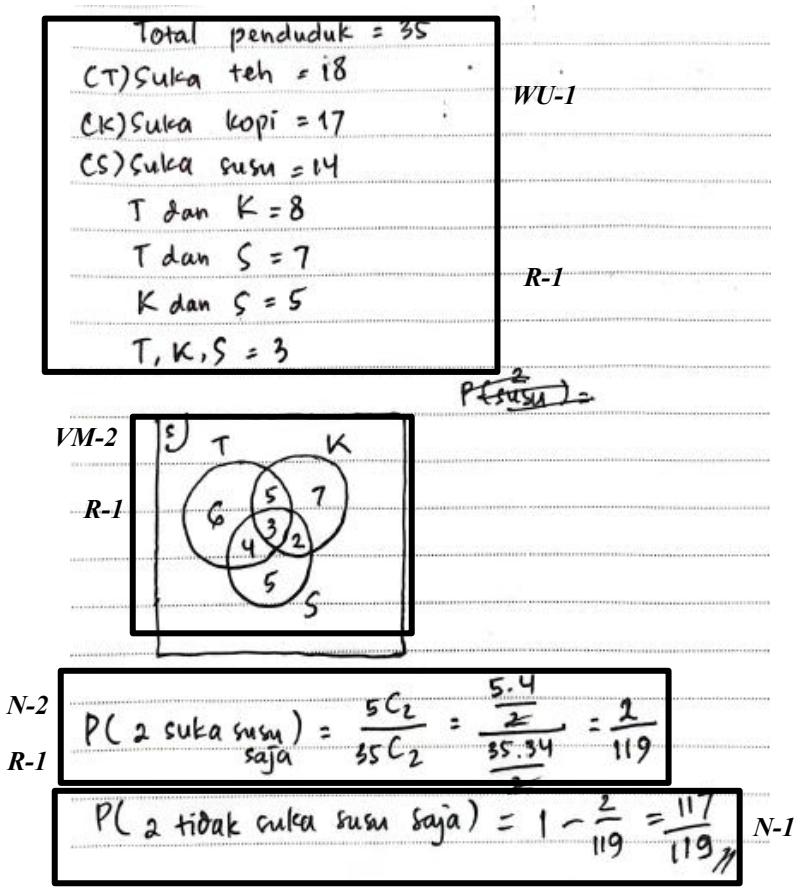

Figure 3. Subject 2 Work Results (S2)

It can be inferred from Figure 3 that the person successfully completed the task. The following cognitive review was used to assess the clarification of S2's response from a semi-structured interview between the researcher (P) and the second subject (S2).

P : Give a detailed explanation of your work. (referring to the work of your master)

S2 : All OK, sir. I started by writing down what I learned after reading and comprehending the issue. What is the number of individuals present? Tea, coffee, milk, tea and coffee, tea and milk, coffee and milk, how many people like these? To make it easier to understand, I then drew it in a Venn diagram and finished it using the procedures I was familiar with.

P : So, how are the probability determined? Describe!

S2 : All right, sir. I remembered the method for calculating the likelihood of eating two at once, which is to combine them, because there are five people who favor milk and nothing else. The likelihood value was the outcome of this. I then remembered that the probability of an occurrence and its complement is equal to one. I therefore solved the issue and came to a decision using this rule.

The commognitive components and their descriptions are shown in Table 1 below, which is based on the findings of the study and research participant interviews.

Table 1. Commognitive Components

Commognitive Components	Subcomponents	Descriptions	Coding	Examples
Words use (WU)	*Colloquial*	Utilizing both written and spoken words and sentences in daily life.	WU-1	Probability is a possibility or chance.
	Literate	Utilizing mathematically based phrases, statements, signs, symbols, and numbers.	WU-2	The probability of event A, the number 4, or six appearing
Visual Mediators (VM)	*Concret*	Solving issues with actual stuff.	VM-1	Representing a spherical item with a plastic ball
	Iconic	Solving problems involves using images or depictions of tables, objects, and other items.	VM-2	Utilizing a Venn diagram to illustrate an issue
	Symbolic	Use variables or symbols to solve issues	VM-3	$P(A)$ represents the likelihood of event A.
Narratives	*Substantiation*	Utilizing logic and justification to address issues.	N-1	$P(A) = 0$, because event A is impossible
	Memorization	Use rules or formulas to address issues.	N-2	$P(A) = \frac{\mathrm{n(A)}}{\mathrm{n(S)}}$
Routines	*Ritualized*	Using previously implemented steps or patterns.	R-1	Determine the number of members of A or B using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
	Exploratory	Using new ways to solve problems.	R-2	Determine the number of students who do not like to participate in two activities using the formula $n(A \cap B)^c = n(S) - n(A \cup B)$

3.2. Discussion

The participants in this study were able to accurately read and comprehend the information in the problem text, which allowed them to put it down in a mathematical statement. The precision of the written mathematical statement depends on a thorough comprehension of the problem (Erikson & Erikson, 2019; Larsson, 2017). A thorough conceptual grasp of mathematical concepts is demonstrated by the ability to express mathematical phrases accurately (Jones, 2017; Widana, 2018). This is in accordance with the view that mathematical statements demonstrate mastery of a mathematical idea (Andersson & Palm, 2017; Kyaruzi, Strijbos, Ufer, & Brown, 2018). Along with having a solid conceptual understanding, the research participants were also able to fully and successfully use pictures to illustrate common statements in the questions. According to Fajriyah et al. (2019), adequate illustrations show strong conceptual grasp, which supports this finding. Additionally, the subjects implemented routine tasks correctly, including choosing formulas, identifying crucial information, and methodically carrying out procedural steps. This supports the assertion Viirman (2015) that if the problem-solving steps have been completed successfully, the routine procedures will be carried out correctly.

The student successfully came to a reasonable conclusion by applying the probability formula for compound events using a combination technique. This is consistent with the viewpoint of Panadero, Jonsson, & Botella (2017); Rakoczy,dkk. (2019); Roth, Ogrin, & Schmitz (2016) that the stated conclusions have benefited from the previously gained comprehension of the idea of probability. Until they came up with the right answer, the subjects followed the solution stages precisely and consecutively. This result supports the researcher's viewpoint. Nardi, Ryve, Stadler, & Viirman (2014) which asserts that the topic has communicated logically by solving the problem step-by-step until the ultimate solution is found when the problem-solving process is documented in a sequential manner. The subject considers the outcomes derived from successfully executed steps when making inferences. This is consistent with the viewpoint of Roberts & le Roux (2018) which states that after presenting their arguments, individuals go through a solution process based on actions conducted in line with the question's contents or by consulting earlier actions. Ultimately, the participants arrive at a clear and unambiguous solution to the problem. This result is consistent with the viewpoint of Nachlieli & Heyd-Metzuyanim (2021) and Viirman & Nardi (2019b) that uncertainty won't arise from conclusions drawn from the outcomes of proper and sequential steps.

4. CONCLUSION

The purpose of this study is to characterize how first-year students' work in addressing probability issues involves cognitive processes. The findings show that students generally possess sufficient proficiency in handling probability problems. The material in the challenge can be written by students in simple, understandable terms (word use). With the help of pictures (visual mediators), students can accurately depict the information in the problem. Students can utilize mathematics to accurately calculate the probability of an event and develop inferences (narratives) based on their prior knowledge (routines). However, more studies on students studying other mathematical subjects, like combinatorics and mathematical logic, are required.

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