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UNVEILING HIDDEN MISCONCEPTIONS: A NEW PERSPECTIVE ON FIRST-YEAR STUDENTS' ALGEBRAIC UNDERSTANDING

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ABSTRACT

This study aims to uncover and analyze hidden misconceptions in algebraic understanding among first-year students in the Mathematics Education Department at a private university in Malang. Conducted from January to March 2024, the research involves two first-year students as subjects. The first subject exhibits the misconception known as "illegal canceling," while the second subject demonstrates the "assumption of distributivity of exponents." Utilizing a qualitative research methodology, diagnostic interviews and problem-solving tasks were employed to identify and understand these misconceptions. Our findings indicate that these hidden misconceptions significantly hinder the students' ability to correctly solve algebraic problems and understand underlying concepts. For the first subject, the error in illegal canceling often led to incorrect simplifications, while the second subject's erroneous application of exponent rules resulted in fundamentally flawed solutions. By systematically categorizing these misconceptions, the study offers tailored instructional strategies to effectively address and correct them. This research highlights the critical need for early detection and targeted intervention in mathematical education. By providing a deeper insight into specific algebraic misunderstandings, the study suggests practical approaches for educators to enhance their teaching methodologies, ultimately improving students' overall algebra comprehension. The novelty of this research lies in its detailed focus on hidden misconceptions and the provision of actionable solutions, offering a fresh perspective on improving algebra education for first-year university students.

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1. INTRODUCTION

Algebra, as a foundational component of mathematical education, plays a crucial role in shaping students' problem-solving skills and logical reasoning abilities (Aygör & Ozdag, 2012). However, despite its significance, many students encounter challenges in mastering algebraic concepts, often due to underlying misconceptions that go unnoticed. In the realm of mathematical education, understanding and addressing these hidden misconceptions are paramount for fostering a solid grasp of algebra among students, particularly those in their first year of university studies.

Previous research has highlighted the prevalence of misconceptions among students and emphasized the importance of early detection and intervention to address these issues effectively. By building upon this theoretical foundation, the current study seeks to contribute to the growing body of knowledge regarding hidden misconceptions in algebraic understanding among first-year university students (Aquino et al., 2017; Aygör & Ozdag, 2012; Booth et al., 2016; Jian-xin, 2013; Wilkie, 2016; Zorn, 2012).

The primary objective of this research is to uncover and analyze hidden misconceptions in algebraic understanding among first-year students. Specifically, the study aims to identify common misconceptions that hinder students' ability to solve algebraic problems accurately and comprehend underlying concepts. By achieving this objective, the research aims to provide insights into the nature of these misconceptions and offer tailored instructional strategies to address them effectively.

Misconceptions in algebra can significantly impede students' progress in understanding and solving mathematical problems. One common misconception is the misunderstanding of the distributive property. Some students may incorrectly assume that multiplication distributes over addition or subtraction in all cases, leading to errors in simplification or expansion of algebraic expressions. For instance, a student might erroneously believe that $a\ (b + c)$ is equal to $ab + ac$ in all situations, neglecting the importance of parentheses and the order of operations.

Another prevalent misconception involves the mishandling of negative numbers. Students often struggle with operations involving negative integers, particularly in equations or expressions where negative signs are distributed. For example, they may incorrectly simplify $-2x - 3y$ as $-5xy$, neglecting to distribute the negative sign to both terms. This misconception can lead to errors in calculations and a fundamental misunderstanding of the arithmetic rules governing negative numbers (Godden et al., 2013).

Furthermore, misconceptions regarding exponents and their properties are widespread among algebra students. For instance, some students mistakenly believe that adding exponents is equivalent to multiplying the bases, leading to errors in simplifying expressions involving exponentiation. Additionally, students may struggle with negative exponents, often misinterpreting them as reciprocals rather than understanding their true meaning as indicating division by the base raised to a positive exponent (Parwati & Suharta, 2020).

A common algebraic misconception relates to the concept of variables. Students may struggle to grasp the notion of variables representing unknown quantities and instead treat them as fixed values. This misunderstanding can hinder their ability to solve equations and translate real-world problems into algebraic expressions effectively. For instance, students may incorrectly interpret the expression $2x + 3$ as representing a fixed value rather than a variable quantity multiplied by two (Luneta & Makonye, 2010; Nadira et al., 2023).

Lastly, misconceptions regarding algebraic operations such as canceling terms can lead to errors in simplification and solution processes. Students may wrongly cancel terms in algebraic expressions, especially when dealing with fractions or complex equations. For instance, they may erroneously cancel out terms in an equation without considering the conditions under which such operations are valid, leading to incorrect solutions and a flawed understanding of algebraic manipulation principles. Overall, addressing these common misconceptions is crucial for fostering a deeper understanding of algebra among students (Faradiba, 2019, 2022; Faradiba & Alifiani, 2020; Nisa & Faradiba, 2023; Susanti & Faradiba, 2022; Widdah & Faradiba, 2022).

This research holds significant implications for both theoretical understanding and practical application in the field of mathematical education. By uncovering hidden misconceptions among first-year students, the study contributes to a deeper understanding of the challenges faced by students in mastering algebraic concepts. Furthermore, by offering tailored instructional strategies to address these misconceptions, the research provides practical guidance for educators seeking to improve their teaching methodologies and enhance students' algebra comprehension. Overall, the study underscores the critical importance of early detection and targeted intervention in addressing hidden misconceptions, thereby promoting more effective mathematics education for first-year university students.

2. METHOD

To address the research objectives, a qualitative research approach was adopted (Creswell & Guetterman, 2019), incorporating a combination of diagnostic interviews and problem-solving tasks designed to comprehensively explore students' cognitive processes and reasoning patterns. This methodological framework was selected to facilitate the identification and detailed analysis of students' concealed misconceptions, particularly those related to fundamental algebraic concepts. Two first-year students from the Mathematics Education Department at Universitas Islam Malang were purposefully selected as primary participants for an in-depth case study. These students were chosen based on their demonstrated difficulties and errors in comprehending algebraic principles, which were revealed through preliminary assessments. Each participant exhibited distinct misunderstandings that warranted focused investigation, providing a unique window into how misconceptions manifest and influence mathematical reasoning.

The research involved extensive and iterative diagnostic interviews, during which students engaged in problem-solving tasks related to algebraic concepts, such as systems of linear equations, matrices, and determinants. These sessions were designed to probe students' thought processes, enabling the researchers to track the formation and persistence of misconceptions. Furthermore, interviews allowed the exploration of the underlying cognitive frameworks and reasoning strategies that contributed to these errors. Additionally, instructional sessions were conducted with a broader group of 15 first-year Mathematics Department students at Universitas Islam Malang, covering essential topics like matrices, determinants, and systems of linear equations. The instructional component aimed to create a shared learning experience, providing a backdrop for observing the emergence of common misconceptions.

The primary goal of this comprehensive research effort was to not only pinpoint prevalent misunderstandings but also analyze their origins and implications on students' overall grasp of algebraic concepts. This analysis would inform targeted instructional strategies, aiming to mitigate these misconceptions and improve students' algebraic reasoning abilities.

The data derived from these observations were juxtaposed with the test results. Subsequently, attention was directed towards the main area of focus derived from the observations: the identification of misconceptions demonstrated by students during problem-solving exercises. In this study, subjects were given the following questions to obtain data.

If $A = \begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix}$ *and* $A^2 = PA + QI$ *(I is the identity matrix), determine the values of* P *and* Q!

3. RESULTS AND DISCUSSION

3.1 Result

The misconceptions that the students had have been scanned without any changes. The answer sheets of different students' misconceeptions are below. Figure 1 presents the work of Subject 1 (S1). S1 addressed a problem involving matrix multiplication, where the result of A squared is obtained by multiplying matrix A by itself. S1 exhibited a misconception in performing matrix multiplication. Instead of correctly multiplying the matrices, S1 directly squared the individual elements of matrix A, failing to understand that the process for matrix multiplication requires a different approach. Figure 1 presents the work of Subject 1 (S1). S1 addressed a problem involving matrix multiplication, where the result of A squared is obtained by multiplying matrix A by itself. S1 exhibited a misconception in performing matrix multiplication. Instead of correctly multiplying the matrices, S1 directly squared the individual elements of matrix A, failing to understand that the process for matrix multiplication requires a different approach.

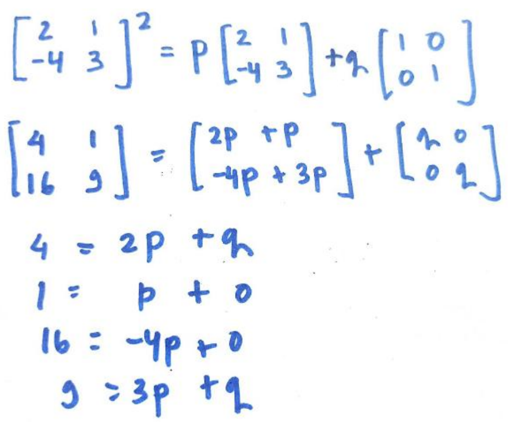

Figure 1. The answer of S1

Figure 2 presents the answer of Subject 2 (S2). The misconception experienced by S2 is interpreting the identity matrix as a matrix with all elements being 1. In fact, an identity matrix should have 1s on the main diagonal and 0s on all other positions. At the conclusion of the solution, S2 arrives at two contradictory equations, namely 5 = p + q and 5 = 3p + q.

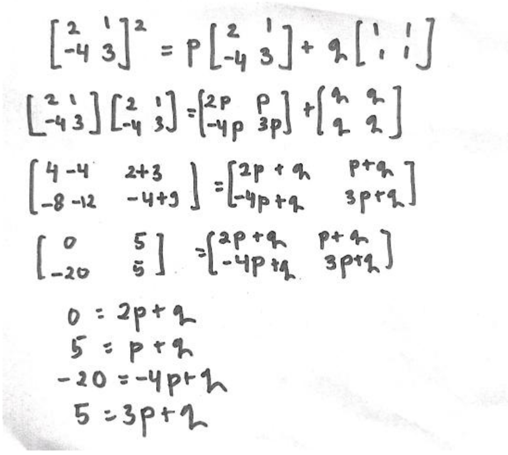

Figure 2. The answer of S2

3.2 Discussion

The findings from Subject 1 (S1) and Subject 2 (S2) provide significant insights into the nature and impact of algebraic misconceptions among first-year students in the Mathematics Education Department. These misconceptions, specifically in the areas of matrix multiplication and identity matrices, highlight critical areas where educational interventions are needed.

S1's work reveals a fundamental misunderstanding of matrix multiplication. Instead of correctly performing the matrix product A × A, S1 simply squared each element within matrix A. This error indicates a lack of comprehension regarding the matrix multiplication process, where each element in the resulting matrix is the sum of products of corresponding elements from the rows and columns of the matrices being multiplied. This misconception demonstrates the necessity for reinforcing the conceptual framework behind matrix operations (Widdah & Faradiba, 2022).

In mathematics education, it is essential for students to grasp the procedural and conceptual differences in operations such as addition, subtraction, and multiplication when applied to matrices versus scalar quantities (Lenz et al., 2020; Saleh et al., 2017). S1's error, where individual elements were squared rather than engaging with the entire matrix structure, underscores a superficial understanding. This suggests that students may benefit from more practical examples and visual aids to illustrate the step-by-step process of matrix multiplication.

S2's response illustrates a different yet equally impactful misconception. By interpreting the identity matrix incorrectly, S2 showed a fundamental misunderstanding of its definition and properties. An identity matrix, by definition, has ones on the main diagonal and zeros elsewhere. S2, however, interpreted the identity matrix as having all elements equal to one, leading to significant errors in subsequent calculations and conclusions.

This misinterpretation by S2 resulted in the derivation of contradictory equations, $5 = p + q$ and $5 = 3p + q$. These inconsistencies arise from the flawed application of the identity matrix in solving the system of equations. The identity matrix plays a crucial role in linear algebra as it serves as the multiplicative identity in matrix operations. Misunderstanding this concept can lead to fundamental errors in solving linear systems and other advanced topics.

The errors observed in both subjects suggest a need for a more robust instructional approach that emphasizes the underlying principles of matrix operations and the properties of special matrices such as the identity matrix. Interactive learning tools, peer instruction, and formative assessments can be effective strategies to address these gaps. Additionally, integrating real-world applications and problems that require the correct use of these concepts can reinforce their understanding (Agung et al., 2021; Arboledas et al., 2020; Booth et al., 2014; Noutsara et al., 2021; Nurlaeli & Widiyatmoko, 2023; Saleh et al., 2017; Satriani et al., 2020).

Furthermore, it is important to provide continuous and targeted feedback to students. The misconceptions identified in S1 and S2's work were likely not detected and corrected promptly, allowing these errors to persist. Regular diagnostic assessments can help educators identify and address these misconceptions early in the learning process (Godden et al., 2013; Luneta & Makonye, 2010; Mamba, 2013; Royce, 2019).

The identification of these specific misconceptions also suggests the potential benefit of incorporating more detailed discussions and explorations of common errors into the curriculum. By analyzing and understanding why students might develop such misconceptions, educators can better tailor their teaching methods to preemptively address and correct these misunderstandings (Faradiba et al., 2023, 2024; Nadira et al., 2023; Nor et al., 2023; Puspita Negara et al., 2023).

In conclusion, the misconceptions demonstrated by S1 and S2 underline the critical need for a thorough and nuanced approach to teaching matrix operations and the properties of matrices in introductory algebra courses. By focusing on conceptual understanding, providing practical applications, and utilizing continuous assessment and feedback, educators can significantly improve students' comprehension and reduce the prevalence of such fundamental errors. This approach not only enhances the learning experience but also prepares students for more advanced mathematical challenges.

4. CONCLUSION

This study meticulously identified and analyzed specific errors in understanding matrix multiplication and the properties of the identity matrix, as demonstrated by the two subjects. These findings reveal that despite prior instruction, significant gaps in foundational knowledge persist, necessitating targeted educational interventions. Subject 1 (S1) exhibited a fundamental misunderstanding of matrix multiplication, where the student incorrectly squared individual elements instead of performing the correct matrix product. This misconception highlights a superficial grasp of the matrix operation process, suggesting that more robust instructional techniques and practical demonstrations are needed to reinforce students' conceptual understanding. By focusing on the procedural nuances and the reasoning behind matrix operations, educators can help students build a stronger foundation in algebra. Subject 2 (S2) displayed a critical error in interpreting the identity matrix, treating it as a matrix with all elements equal to one instead of recognizing its proper form with ones on the main diagonal and zeros elsewhere. This misunderstanding led to incorrect and contradictory solutions when solving systems of linear equations. This finding underscores the importance of clearly conveying the properties and roles of special matrices in algebra, as well as integrating consistent practice and feedback to ensure these concepts are correctly understood and applied. Overall, this study emphasizes the necessity for early detection and correction of algebraic misconceptions. By adopting a more comprehensive and interactive approach to teaching these fundamental concepts, educators can significantly improve students' understanding and prevent the propagation of these errors. The research advocates for continuous assessment, practical applications, and tailored instructional strategies to address and rectify misconceptions effectively. Ultimately, these efforts will enhance students' overall algebraic proficiency and better prepare them for advanced mathematical challenges.

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