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ANALYZING STUDENTS' ABILITIES AS PROSPECTIVE TEACHERS OF MATHEMATICS IN CONSTRUCTING PROOFS

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ABSTRACT

Mathematical proof demands accuracy and precision in formulating correct and logical arguments. Students need to develop their ability to produce accurate and precise proofs. This study aims to analyze the ability of prospective mathematics teachers in constructing geometry and algebraic proofs. The research subjects were 8 prospective mathematics teacher students, four male and four female. This descriptive qualitative research begins with giving tests to research subjects, and then we conduct interviews. The results showed that male students were better at compiling geometry than algebraic proofs. At the same time, female students are better at compiling algebraic proofs than in geometry. This result is due to the spatial ability of men better than women. When compiling geometry proofs, apply procedural, syntactic, and semantic proofs. When compiling algebraic proofs, only apply procedural proofs. Female students, when compiling geometry proofs, only apply procedural proofs. When compiling algebraic proofs, they apply procedural proofs and syntactic proofs.

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1. INTRODUCTION

The development of mathematics encourages humans to be more creative in applying mathematics in everyday life. This application of mathematics cannot be separated from the ability to solve problems that require an understanding of concepts. On the other hand, mathematical concepts can be obtained or developed through mathematical proof. The ability to construct logical and structured proofs is a key indicator in understanding and mastering mathematical concepts in depth. However, in practice, many students face difficulties compiling good and correct proofs, both in geometry and algebra (Oxburgh, 2021). In mathematical epistemology, proof is considered as the main foothold for building valid mathematical knowledge. The construction of proof involves a deep understanding of mathematical concepts, consistent use of logic, and critical abstract thinking. Mathematical knowledge is considered an objective reality that exists beyond individual perception. This implies that students' ability to construct proofs depends not only on their subjective views but also on an objective understanding of mathematical concepts and the applicable rules of logic (Moore, 2016). The construction of mathematical proofs involves complex cognitive processes. Students need to organize and connect mathematical concepts, apply correct logical principles, and develop the necessary abstract thinking.

When we construct mathematical proofs it does not always go smoothly, mistakes are common (Schlesinger et al., 2018). However, these mistakes can be opportunities to improve understanding and enrich knowledge. In improving understanding and enriching knowledge, students can collaborate with others Interaction with peers, group discussions, and feedback from others can help students deepen their understanding of mathematical proofs. Constructing proofs also includes collaborative and social aspects of learning mathematical proofs. Thus, student teachers can engage their understanding of the nature of mathematical knowledge, the process of proof construction, errors and revisions, and social aspects when constructing proofs (Abbott, 2013).

The mathematical realism perspective considers mathematical entities such as numbers, patterns, and mathematical structures, to have an objective existence that is independent of the human mind (Font et al., 2013). Ontologically, research on students' ability to construct proofs will pay attention to students' relationship with mathematical entities and how they represent these entities in the proof process. From a constructivist perspective, mathematics emphasizes the role of construction and individual mental activity in the formation of mathematical knowledge. The Platonism perspective considers that mathematical entities are abstract objects that exist independently and universally (Font et al., 2013). Students interact with these mathematical subjects.

The structuralist perspective emphasizes the importance of structure and pattern relationships in mathematics. Students recognise, understand, and use mathematical structures in constructing proofs, both in geometry and algebra. Research on analysing students' ability to construct mathematical proofs will gain a deeper understanding of the nature of the mathematical entities involved, the relationships between mathematical concepts, and the role of human thinking in the proof process (Feriyanto, 2018).

Mathematical proof demands accuracy and precision in formulating correct and logical arguments. Students need to develop their ability to produce accurate and precise proofs (Cheng, 2017). Mathematical proofs must be able to be structured in a way that is clear and can be understood by others. Students' ability to construct proofs that can be communicated well is important in the context of axiology. Mathematical proof can also be considered as an art form that shows beauty and harmony in the construction of arguments and relationships between mathematical concepts. The process of mathematical proof involves critical thinking and reflection on the concepts and arguments used. Research can involve evaluating students' ability to analyse and evaluate the mathematical proofs they encounter, and to identify weaknesses or errors in those proofs.

Mathematical proof is closely related to geometry and algebra. Geometry and algebra complement each other and help in understanding mathematical concepts thoroughly (Usiskin, 2021). This interrelationship allows for the simultaneous use of geometry and algebraic approaches in solving mathematical problems, modelling real-world situations, and developing a deeper understanding of mathematical concepts. Mathematical proofs often require perseverance and creativity in finding new approaches and innovative solutions. Students' ability to construct mathematical proofs can involve a deeper understanding of the values associated with mathematical proofs and how they can be applied in learning and assessment contexts (Kelley & Knowles, 2016).

The problem in this study is how the ability of prospective mathematics teachers in preparing geometry and algebra proofs. Some of the problems that may arise include: difficulty in identifying the right steps to construct a proof; lack of understanding of the structure of the proof and how to organise arguments logically; difficulty in understanding the mathematical concepts underlying the proof; lack of ability in connecting different mathematical concepts to construct a consistent proof; and difficulty in applying abstract thinking in constructing mathematical proofs (Stylianides et al., 2016) .

This study aims to describe the ability of prospective mathematics teachers in constructing geometry and algebraic proofs based on the problems faced by students in constructing mathematical proofs and identifying factors that affect their ability. This article can be a new insight in the context of teaching mathematics because it discusses how prospective teachers integrate the concepts of geometry and algebra in preparing proofs.

2. METHOD

Our research is qualitative descriptive research. We describe the ability of prospective mathematics teacher students in preparing geometry and algebraic proofs. The natural subjects of the research were 8 mathematics teacher candidates. Four male and four female students. We asked them to construct one geometry proof and one algebra proof. We provided two geometry problems and two algebra problems, but students were asked to do one problem each. The proof we provided are as follows.

1. Prove that the product of two consecutive integers is always divisible by 2
2. Prove that the product of three consecutive integers is always divisible by 6

Figure 1. Algebra Problem

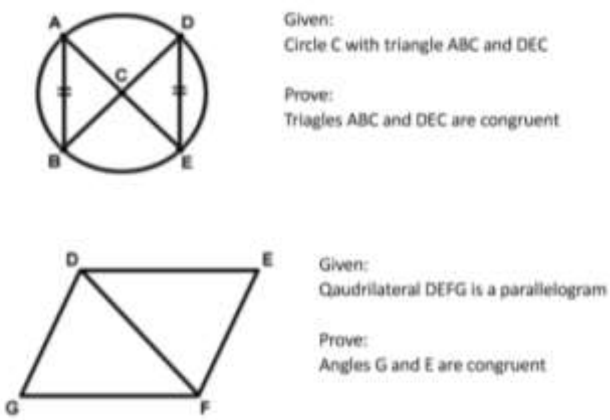


Figure 2. Geometry Problem

We interviewed the subjects using semi-structured interviews. We did it after the subject finished working on geometry and algebra problems. Next, we analyzed the results of compiling evidence and interviews based on the process of compiling evidence. The way we analyzed the data was by reducing, presenting data, and making conclusions so that we obtained an analysis of the ability of prospective mathematics teachers in constructing the proof. Here is the process of constructing proofs that students can use (Weber et al., 2014).

Table 1. The Constructing Proofs Process

Process	Aspects
Procedural Proofs	Try to build a proof by applying the procedure
	Define a specific set of steps
Synthetic Proofs	Manipulating definitions
Semantic Proofs	Trying to understand why a question is correct by checking the representation

3. RESULTS AND DISCUSSION

3.1. Results

In this section, we describe the research data based on tests and interviews with 8 prospective mathematics teacher students. We gave students the task of constructing proofs related to geometry and algebra material. In the first part, we describe the students' work in constructing geometry proofs. This study was designed to investigate the students' proof ability. First, we analysed four male subjects' proofs in algebra.

Table 2. Analysis of Male Subjects in Constructing Proofs of Algebraic

Subject	Result	Analysis
AA	AA wrote an example of 3 positive numbers that when added together are divisible by 6. Suppose the numbers are 2, 3, and 4. Furthermore, he wrote "So, the statement is True".	AA attempts to construct proof by applying a procedure (applying procedural evidence)
MH	MH denote two numbers by a and a+1, and suppose a is an even number then it must be divisible by 2 because it is 2. Suppose 0, 2, 4, ... etc. are divisible by 2 or -2, -4 are also divisible by 2.	MH tries to construct proof by applying a procedure (applying procedural proof)
BD	BD wrote that two integers are divisible by 2 if the last number is divisible by 2. Suppose the integers are 3 and 2 then 3 x 2 = 6, since 2|6	BD tries to build evidence by applying a procedure (applying procedural proof)
MR	MR menuliskan contoh 3 bilangan positif yang jika dijumlahkan habis dibagi 6. Kemudian menuliskan n(n+1)(n+2) habis dibagi 6.	MR wrote down an example of 3 positive numbers that when added together are divisible by 6. Then wrote n(n+1)(n+2) is divisible by 6.

From table 2, it can be seen that all four subjects applied procedural proofs, i.e. tried to build evidence by applying procedures. They determined a specific set of steps they believed would result in a valid proof. It is possible that the procedure is meaningful to the proof. They understand why successful application of the procedure will result in a logical argument and establish the truth of the claim to be proved. H ere are MR and MH's answers and our interviews with them.

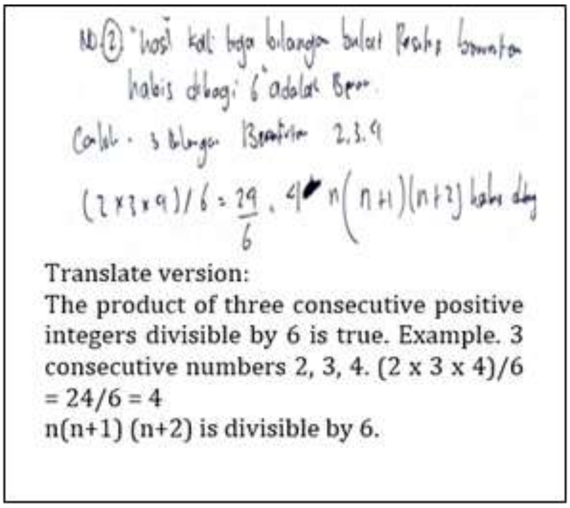


Figure 3. Subject MR's Answer

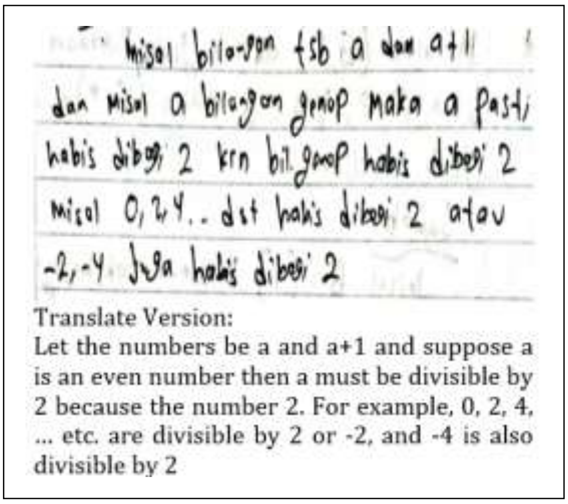


Figure 4. Subject BD's Answer

From both pictures, we can see that they only applied procedural proof. But what they have done has not applied the steps of proof that have been taught. Although they realize that constructing proof requires logical thinking. This is implied in the following interview results.

P	:	*How do you understand the concept of mathematical proof?*
MR	:	*In my opinion, the mathematical proof is work that requires convincing the correctness of a theorem.*
P	:	*How do you see the role of logic in constructing mathematical proofs?*
MH	:	*Very important, because obtaining valid evidence must involve logical thinking*

We do not present the work and interviews of the other two subjects, because their work and interviews are almost the same as those of MR and BD. In the second part, we analyzed four male subjects in constructing geometry proofs.

Table 3. Analysis of Male Subjects in Constructing Proofs of Geometry

Subject	Result	Analysis
AA	AA write "Find $AB \cong DE$, hence $\overline{AC} = \overline{CD}, \overline{CB} = \overline{CE}, \angle CAB = \angle CDE, \angle CBA = \angle CED, \angle ACB = \angle DCE, \overline{AE} = \overline{DB}$, SO $\Delta ABC \cong \Delta DEC$"	AA attempts to construct evidence by applying a procedure (applying procedural proof), manipulating definitions (applying syntactic proof), and examining representations (applying semantic proof).
MH	MH wrote "Based on the properties of a parallelogram: $\angle G = \angle D = \angle E = \angle F = 90^o$, jadi $\angle G =$	MH attempted to apply procedural evidence by trying to construct evidence by applying a procedure

Subject	Result	Analysis
	$\angle E\ (same), \angle G = \angle E\ (retrieved\ from), \angle G = \angle G\ (kongruen)$"	despite making mistakes
BD	BD only wrote down the angles but not clearly enough	BD does not apply any proof
MR	MR write "$AB \cong DE$, hence $\overline{AC} = \overline{CD}, \overline{CB} = \overline{CE}, \angle CAB = \angle CDE, \angle CBA = \angle CED, \angle ACB = \angle DCE, \overline{AE} = \overline{DB}$, So $\Delta ABC \cong \Delta DEC$"	AA attempts to construct proof by applying a procedure (applying procedural proof), manipulating definitions (applying syntactic proof), and examining representations (applying semantic proof).

Based on table 3. it can be seen that subjects AA and MR applied procedural evidence, i.e. tried to build evidence by applying procedures. They determine a specific set of steps they believe will produce a valid proof. It is possible that the procedure is meaningful to the proof. They understand why successful application of the procedure will result in a logical argument and establish the truth of the claim to be proved. They also apply syntactic proofs, i.e. trying to write proofs by manipulating correctly stated definitions and other logically relevant facts. Finally, they applied semantic proofs, i.e. trying to understand why a statement is true by examining representations (e.g. diagrams) of relevant mathematical objects and then using these intuitive arguments as a basis for constructing formal proofs. The following are AA and MR's answers and the results of our interviews with them.

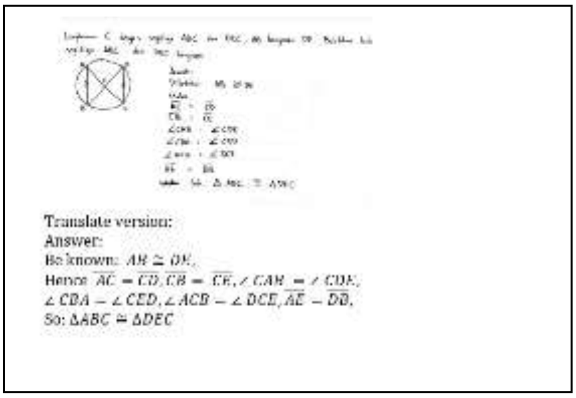


Figure 5. The answer of subject AA

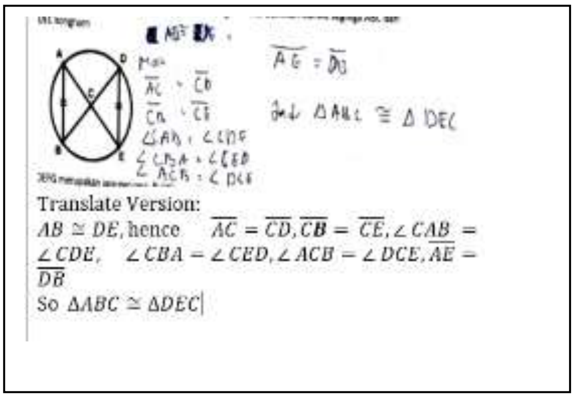


Figure **6**. The answer of subject MH

From the two pictures, we can see that they only applied procedural proof, syntactic proof, and semantic proof. Although they stated that they had experienced difficulties when compiling the Proof but, in the end, they could overcome the difficulties experienced because they already had the right technique or strategy. They have also ensured that the evidence compiled is correct and precise. They can also develop the ability to compile evidence. This is implied in the following interview results.

P : *What obstacles or difficulties do you face when trying to construct mathematical proofs?*

AA : *I have trouble finding the right ideas or concepts to*

		prove my point.
P	:	*How did you overcome these difficulties or obstacles?*
MR	:	*By remembering the theorems that have been taught*
P	:	*What techniques or strategies do you usually use in constructing mathematical proofs?*
AA	:	*Modelling and analogising with existing theorems*
P	:	*How do you ensure that the evidence you compile is correct and complete?*
MR	:	*By looking back at the proofs, I have obtained*
P	:	*How have you developed your mathematical proof building skills over time?*
AA		*With more reading and practice compiling evidence*

We do not present the work and interviews of the other two subjects, as the work does not fulfil the requirements of the proof. In the third part, we analysed four female subjects in constructing proofs of algebraic.

Table 4. Analysis of Female Subjects in Constructing Proofs of Algebraic

Subject	Result	Analysis
AR	AR write $(n-1)(n)(n+1) = n^3n + 3n^2 + 3n = 3n^2 + 3n + 3(n^2 + 1)$	AR mencoba untuk membangun bukti dengan menerapkan sebuah prosedur (menerapkan bukti prosedural) meskipun tidak sampai pada hasil akhir
IK	IK wrote "if we assume a number is x and a number divisible by 3 will have a remainder of 0,1, or it could be 2. Example of three consecutive numbers 2, 3, 4, (2,3,4)/6". So the consecutive numbers in the problem above (2,3,4)/6 = 24/6 = 4	IK tries to build evidence by applying a procedure (applying procedural evidence)
CN	CN wrote that if the integer is even, then it is always divisible by 2, if the integer is odd, suppose a = odd integer, 1 = odd integer then a+1 = even number. Even numbers are always divisible by 2	BD tries to construct evidence by applying a procedure (applying procedural evidence)
NS	NS Write down: Two consecutive numbers: n, n+1, 2|n(n+1), 2| $n^2 + n$, forn = 1, $n^2 + n$ so 2|2 so the property 2| $n^2 + n$ is true for n = 1. Suppose the property 2| $n^2 + n$ is true n = k, yaitu 2| $k^2 + k$. it will be proved that the property 2| $n^2 + n$ is true for n = k+1, i.e 2| $(k+1)^2 +$	NS tried to construct a proof by applying a procedure (applying procedural proof), manipulating definitions (applying syntactic proof), and examining representations (applying semantic proof).

Subject	Result	Analysis
	$(k+1)$. Note that $(k+1)^2 + (k+1) = (k^2+2k+1+k+1) = k^2+2k+2 = k^2+k+2(k+1)$. So the property $2 \mid n^2+n$ is true for n = k+1. By mathematical induction, the property $2 \mid n^2+n$ is true for all natural numbers n.	

Based on table 4. it can be seen that three subjects (AR, IK, CN) applied procedural evidence, i.e. tried to build evidence by applying procedures. They determine a specific set of steps they believe will produce a valid proof. It is possible that the procedure is meaningful to the proof. They understand why successful application of the procedure will produce a logical argument and establish the truth of the claim to be proved. Whereas subject NS applied procedural evidence, which is trying to build evidence by applying procedures. He determines a specific set of steps they believe will produce valid evidence. It is possible that the procedure is meaningful to the proof. He understands why successful application of the procedure will produce a logical argument and establish the truth of the claim to be proved. He also applied syntactic proof, which is trying to write a proof by manipulating correctly stated definitions and other logically relevant facts. Finally, NS applied semantic proof, which is trying to understand why a statement is true by examining representations (e.g. diagrams) of relevant mathematical objects and then using these intuitive arguments as the basis for constructing a formal proof. The following are IK and NS' answers and the results of our interviews with them.

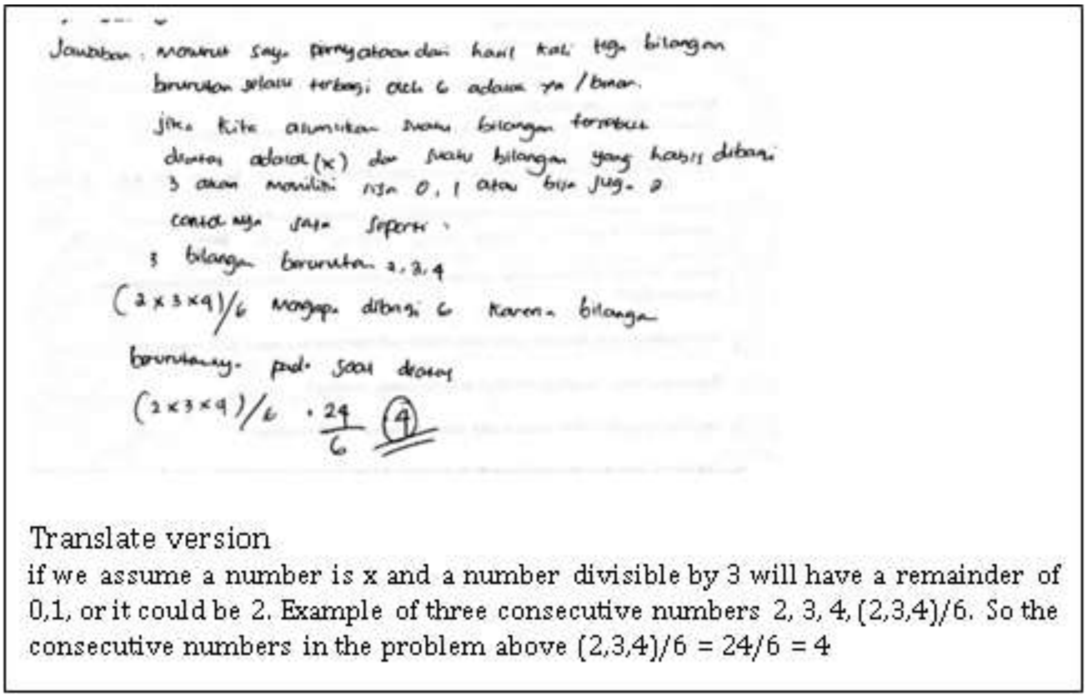


Figure 7. the answer of subject IK

Based on Figure 7 we can see that IK only applied procedural proof. But what he did has not applied the proof steps that have been taught. Even though he realised that compiling proofs requires logical thinking. This is implied in the following interview results.

P	:	*How do you understand the concept of mathematical proof?*
IK	:	*Find the truth of mathematical statements*
P	:	*How do you see the role of logic in constructing mathematical proofs?*
IK	:	*I think it is important to involve logical thinking when compiling proofs.*

Translate Version:

Two consecutive numbers: n, n+1, 2|n(n+1), 2| n^2+n, for n = 1, n^2+n so that 2|2 becomes the property of 2| n^2+n is true for n = 1. Suppose property 2| n^2+n is true for n = k, which is 2| k^2+k. We will prove property 2| n^2+n is true for n = k+1, which is 2| $(k+1)^2$+(k+1). Note that $(k+1)^2$+(k+1) =(k^2+2k+1+k+1) =k^2+2k+2=k^2+k+2(k+1). So, property 2| n^2+n is true for n = k+1. Based on mathematical induction, the property 2| n^2+n is true for all natural numbers n

Figure 8. The Answer of Subject NS

Based on figure 8 we can see that NS tried to construct the proof by applying a procedure (applying procedural proof), manipulating the definition (applying syntactic proof), and checking the representation (applying semantic proof). NS subject understood the concept of proof and realised the role of logic in proof. This is implied in the following interview results

P	*:*	*How do you understand the concept of mathematical proof?*
NS	*:*	*The concept of mathematical proof is a concept used for the truth of a theorem with the help of logic.*
P	*:*	*How do you see the role of logic in constructing mathematical proofs?*
NS	*:*	*The role of logic in proof: making reasonable arguments, making logical connections between concepts and facts, conjecturing and testing based on reason, solving mathematical problems rationally.*

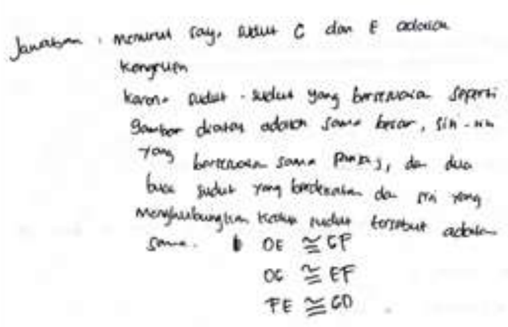
Based on an interview, NS understood the importance of logical thinking in constructing proofs, namely making reasonable arguments, making logical connections between concepts and facts, conjecturing and testing based on reason, and solving mathematical problems rationally.

In the fourth section, we present our analysis of four female subjects in constructing geometry proofs.

Table 5. Analysis of Female Subjects in Constructing Geometry Proofs

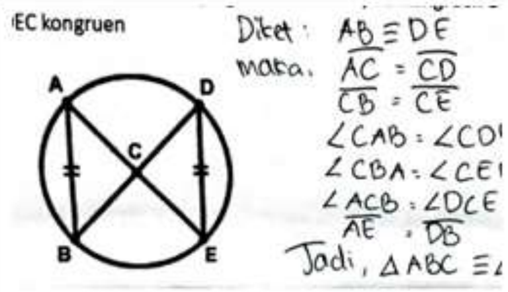
Subject	Result	Analysis
AR	AR did not write anything down	AR does not apply any proof
IK	MH wrote "I think angles C and E are congruent, because the corresponding angles are equal in magnitude, the corresponding sides are equal in length, and the two adjacent angles of the side connecting the two angles are equal.. $DE \cong GF, DG \cong EF, EF \cong GD$	IK attempted to apply procedural evidence by trying to construct evidence by applying a procedure despite making mistakes
CN	CN wrote "Known: $AB \cong DE$, $Hence\ \overline{AC} = \overline{CD}, \overline{CB} = \overline{CE}, \angle CAB = \angle CDE$, $\angle CBA = \angle CED, \angle ACB = \angle DCE, \overline{AE} = \overline{DB}$, So: $\Delta ABC \cong \Delta DEC$	CN attempts to construct evidence by applying a procedure (applying procedural evidence), manipulating definitions (applying syntactic evidence), and examining representations (applying semantic evidence).
NS	NS write "$GR \cong DE$, GD $\cong RE$, DF is the diagonal of the plane, so $DF \cong FD$. Thus, the corresponding side of ΔGFD and ΔEDF same length. Jadi ΔGFD and ΔEDF is congruent"	NS tries to construct proof by applying a procedure (applying procedural proof), manipulating definitions (applying syntactic proof)

Based on table 5. it can be seen that subject IK tried to apply procedural evidence by trying to build evidence by applying a procedure despite making mistakes. CN tried to build evidence by applying a procedure (applying procedural evidence), manipulating definitions (applying syntactic evidence), and checking representations (applying semantic evidence). NS tried to construct the proof by applying a procedure (applying procedural proof), manipulating the definition (applying syntactic proof) despite adding brau dots which were not needed. Here are IK and CN's answers and the results of our interviews with them.



Translate version:
Answer:
I think angles C and E are congruent, because the corresponding angles are equal, the corresponding sides are the same length, and the two adjacent angles of the side connecting the two angles are equal. DE≅GF, DG≅ EF, EF≅GD

Figure 9. The Answer of Subject IK



Translate Version:
Note $AB \cong DE$,
So $\overline{AC} = \overline{CD}, \overline{CB} = \overline{CE}, \angle CAB = \angle CDE$,
$\angle CBA = \angle CED, \angle ACB = \angle DCE, \overline{AE} = \overline{DB}$
Then $\Delta ABC \cong \Delta DEC$

Figure 10. The Answer of Subject CN

From the two pictures, we can see that they only applied procedural proof, syntactic proof, and semantic proof. Although they stated that they had difficulties when compiling the evidence, they were able to overcome the difficulties experienced because they already had the right technique or strategy. They have also ensured that the evidence compiled is correct and precise. They can also develop the ability to compile evidence. This is implied in the following interview.

P	:	*What obstacles or difficulties do you face when trying to construct mathematical proofs?*
IK	:	*Sometimes find it difficult to think logically about new or unprecedented proofs*
P	:	*How did you overcome these difficulties or obstacles?*
NC	:	*Seek information from various sources (learning) and learn more specifically about how to compile evidence.*
P	:	*What techniques or strategies do you usually use in constructing mathematical proofs?*
IK	:	*Try to remember the theorems learnt in the past*
P	:	*How do you ensure that the evidence you compile is correct and complete?*
NC	:	*By proving not just once and looking back at the proof constructions I have done*
P	:	*How have you developed your mathematical proof building skills over time?*
IK		*Try more exercises*

From the interview results, it was seen that they had difficulty thinking logically, to overcome it by learning from various sources. They recalled the theorems they had learnt to construct the proof. They make sure that the proof construction is correct by rechecking the results obtained. They tried many exercises to develop the ability to construct proofs.

3.2. Discussion

Based on the analysis we have done, male prospective mathematics teacher students in preparing algebraic proofs only apply procedural proofs, namely trying to build proofs by applying procedures. They determine a specific set of steps that they believe will produce a valid proof. Such proof is called pure formal deduction (Bosque et al., 2017). Whereas when constructing geometry proofs, in addition to applying procedural proofs, they apply syntactic proofs, which are trying to write proofs by manipulating correctly stated definitions and other logically relevant facts. This kind of thing is often referred to as proofs based on main ideas (Raman, 2003). In addition, they also apply semantic proofs, i.e. trying to understand why a statement is true by examining representations (e.g., diagrams) of relevant mathematical objects and then using these intuitive arguments as a basis for constructing formal proofs. Composing such proofs is often called proofs that follow intuitive thinking (Weber et al., 2014).

Female prospective mathematics teacher students in preparing algebraic proofs mostly apply procedural proofs. It is possible that the procedure is meaningful to the proof. They understand why successful application of the procedure will produce a logical argument and establish the truth of the claim to be proved. There are students who apply syntactic proofs, which is trying to write proofs by manipulating correctly stated definitions and other logically relevant facts. In addition, she also applied semantic proof, which is trying to understand why a statement is true by examining representations of relevant mathematical objects. This can happen due to several factors, such as individual preferences in learning mathematics, previous learning experiences, interest and aptitude in learning mathematics. Someone who has a strong interest in mathematical concepts will be more likely to use syntax or semantics in understanding the basics of mathematics (Mainali, 2021).

Thus, we see that male pre-service teachers are better at constructing geometry proofs than female pre-service teachers. Conversely, female students are better at compiling algebraic proofs than male students. This is in accordance with the statement of (Maccoby & Jacklin, 2016), men and women have different abilities, among others: Men are superior in visual spatial abilities than women;. men dominantly use their spatial abilities while women use less spatial abilities. Spatial ability here is closely related to geometry ability according to the results of research Capraro (2001) 83% of spatial ability affects geometry ability, the remaining 17% is influenced by other factors not studied. They had difficulty in identifying the appropriate steps to construct a proof; lack of understanding of the structure of the proof and how to organise the argument logically; difficulty in understanding the mathematical concepts underlying the proof; lack of ability in connecting different mathematical concepts to construct a consistent proof; and difficulty applying abstract thinking in constructing mathematical proofs. This could be due to a lack of understanding of basic mathematical concepts, lack of practice and experience, or inappropriate teaching methods applied when learning mathematical proofs (Chebet, 2015).

4. CONCLUSION

The ability of prospective mathematics teachers to prepare proofs in this study is explained in three ways: applying procedural, synthetic, and semantic proofs. There are differences in the ability to compile evidence between male and female students. Male students are better at compiling geometry evidence than female students. Conversely, female students have better abilities in terms of compiling algebraic proofs. Of course, this is beyond the causal factors, such as health, readiness to work on problems, and other factors. Conversely, female students have better abilities in terms of compiling algebraic proofs. Of course, this is beyond the causal factors, such as health, readiness to work on problems, and other factors.

Therefore, we suggest future researchers research compiling proofs related to influencing factors in more detail. We also hope prospective mathematics teacher students practice a lot in compiling geometric and algebraic proofs because it can strengthen mathematical concepts.

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