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Student Comprehension of The Concept of a Geometrical Figure: The Case of Straight Lines and Parallel Line

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Abstract

The objective of this paper is to identify some student's conceptions of the straight line and two parallel lines. This will allow us to evaluate the distance that exists between their conceptions and the theory of these concepts. In doing so, we analyzed the students' answers to a questionnaire: the questionnaire concerned the explication of the straight line and two parallel lines. The results indicate that the students have difficulties to produce an acceptable definition of a straight line and two parallel lines. They have difficulties to find appropriate terms to express their comprehension of these concepts. The definitions they produce are ambiguous and seem to be related to the drawings they have encountered in the classroom. Their answers indicate that their comprehension on the straight line and two parallel lines seem to be in conflict with the theory of these concepts.

Keywords: Concept image, concept definition, straight line, parallel lines

A. Introduction

In Cameroon, the activities prescribed by the mathematical official syllabus at the beginning of secondary school mobilize media that promote visual perception as a knowledge construction method. This can generate misconceptions among students (Coppé, Dorier, & Moreau, 2005; Walter, 2001) : this is particularly the case for the straight line, which is only partially represented on media such as paper or a computer screen. It may be difficult for a beginner to distinguish the straight line from the line segment. It is important for students to be able to conceptualize this notion of the straight line which is not facilitated by such activities that encourage empirical manipulation of their drawing.

Previous research has focused on students' comprehension of some geometric objects (Gutiérrez & Jaime, 1999; Vinner, 1983; Zaslavsky & Shir, 2005). This work highlights the students' difficulties in defining and recognizing geometric figures. This is why, in agreement with some researchers (De Villiers, 1998; Freudenthal, 2012), we think that a good adequacy between the students' conceptions and the theory of figures is necessary for a better use of these definitions in situations that convoke them. Thus, the identification of students' conceptions of geometrical objects could make it possible to envisage the interventions with them in order to remove these difficulties. The research question of this study is the following: What are the students' conceptions of the notions of a straight line and two parallel lines? In this way, we will be able to identify the distance that exists between students' conceptions and the formal theory of these objects. On the other hand, we will look for the possible origins of these conceptions.

We have divided our work into two main parts. In the first part, we present the literature review relates to our objective, we present our framework which is based on Vinner's model: concept image concept definition that allows us to describe and interpret the phenomena observed in the student's production about the notions of a straight line and two parallel lines. The second part concerns the methodology which consists of a didactic engineering after this we present a posteriori analyses of the 8th Grade student answers to a questionnaire. The questionnaire focused on students' descriptions of the concepts of the straight line and two parallel lines. Finally, we discuss our result and conclude our research.

B. Literature Review

Drawings are generally used as a basis for the study of figures at the beginning of secondary school. The status and roles given to these drawings will gradually evolve in the learner, as they gradually become associated with graphic representations of figures. Research in didactics of mathematics reports that this transition that occurs in the objects studied in geometry is often at the origin of students' difficulties.

Figure and drawing are two distinct objects in elementary geometry. According to Robert (2003), the figure is an ideal object, that is, an abstract object. It is an intellectual construction as a model of concrete objects in a reality domain, accessible to our perception. As Walter (2001) says, the figure is the basis of geometric thought. It differs from the drawing, which is an economic substitute for it; more precisely, the drawing is a graphic representation of the figure on a medium that symbolizes the plane. Some figures such as the straight line cannot be fully represented on a support. Hilbert proposes that the straight line as well as the point and the plane are primitive objects and does not need to be formally defined in elementary geometry. This proposal does not seem to be shared, since definitions of this concept are present in some textbooks (Njomgang & Tchonang, 2018). In the light of the above considerations, there is a fear that students may construct misconceptions of this concept.

The definition of mathematical objects is a topic which interests researchers in mathematics education (De Villiers, 1998; Gutiérrez & Jaime, 1999; Zaslavsky & Shir, 2005). Zaslavsky and Shir (2005) have studied how students understand the definition in mathematics, and they highlight the fact that students represent the definition of a geometric object in terms of the role it plays in their reasoning. Gutiérrez and Jaime (1999), for their part, were interested in the students' conceptions of the height of a triangle by pre-service primary school teachers, and they observed that the students expressed incoherent concept image about the height of a triangle. They observed that the concepts image of their participant was the same as those of the primary and secondary students on the same concept. One can assume that teachers' conception of a geometrical figure could influence their student's conception of this figure.

Fujita (2012) studied students 'comprehension of the inclusive relationship between quadrilaterals as well as prototypes phenomena. He notes that more than half of the participants in his studies tended to recognize quadrilaterals through prototype examples even when they knew the definition. He believes that this fact is at the source of students' difficulties in recognizing the inclusive relationship between parallelograms. The students' first encounter with geometric figures is with their graphical representations, and they can construct their comprehension of these figures from their interpretations of these graphical representations. This can contribute to the construction of misconceptions tour students about these figures.

C. Theoretical Framework: Concept Image and Concept Definition

Vinner's model: concept image and concept definition provide a useful theoretical framework that can guide the teacher and researcher in understanding the student's mental process (Gutiérrez & Jaime, 1999; Vinner, 1983; Zaslavsky & Shir, 2005). According to this author, there is a conflict between the structure of written and taught mathematics, where the definition introduces the concept, and the cognitive process of concept acquisition.

The concept image is a concept that is used to describe the total cognitive structure of an individual associated with a given concept. It includes all the mental images and properties of the concept, the theorems, and the processes that are associated with it. It may not be consistent and may have aspects that are very different from the formal definition of the concept namely the definition accepted by the mathematical community. The concept image is constructed over the years through experiences of all kinds, changing as the individual goes through his experiences. Moreover, the concept image is evolutive. When a concept is evoked or when we solve a task related to it, our memory is stimulated and something is evoked. What is evoked is rarely the formal definition of the concept alone, but a set of visual representations, images, properties of objects associated with the concept, theorems related to the concept, or experiences. This set constitutes the concept image.

According to Vinner's model, experience as well as examples of figures encountered either in or out of school play an important role in the formation of the concept image. Very often, the examples of the concept provided to students are not varied. Most of them have specific and common visual properties. These examples of figure representations become prototypes and are the only references to the student when he is led to examine a new situation that mobilizes the concept. According to several studies (Gutiérrez & Jaime, 1999; Zaslavsky & Shir, 2005), concepts image constructed by learners differ from formal theory and contain factors that provoke cognitive conflicts. This is the case, for example, of the first encounter with subtraction in mathematics. It may contain seeds of conflict because of the images it reflects to the students: one of the properties retained by students is that "subtraction decreases the initial quantity"; this property is constitutive of the concept image relating to subtraction at a given point in teaching and learning (Tall & Vinner, 1981). However, this element of the concept image can be a source of conflict when it comes to learning new knowledge related to relative numbers.

Many objects encountered in geometry appear in one form or another before they are formally defined, and it is envisaged that for each individual, this may generate a variety of personal mental images of the objects concerned in his cognitive structure. Formal definition in mathematics is defined by Durand-Guerrier, Hausberger and Spitalas (2015) as an open sentence associated with an object property (e.g. being unlimited) or a relationship between objects (being parallel to) that can be satisfied by some objects in a class of objects and not by others. The student can memorize the definition of a geometric object and produce it when requested. This verbal definition that can be memorized and repeated by the student is called by Vinner concept definition. It is a set of words used to specify the concept. The concept definition is related to the concept as a whole. The concept definition is also, according to Vinner, a student's personal reconstruction of a definition. In this particular case, the concept definition, now called a personal concept definition, consists of the words that the student uses to explain his own evoked concept image. From our point of view, the identification of the properties of objects as well as the relations between objects that are contained in the definition of a concept proposed by a student allows us to have an idea about the evoked concept image. Tall and Vinner (1981) think that, when the definition of a concept is given to a student or when it results from his own construction, the student can modify this definition from time to time. As such, a personal concept definition may differ from a formal concept definition.

Zaslavsky and Shir, (2005) propose the following criteria for a good quality of mathematical definition:

- Non-contradictory: The conditions of a definition must coexist. In other words, a definition should not contain contradictory object properties;
- Unambiguous: its meaning must be uniquely interpreted;
- Independent of representations: is not a description of a particular drawing of the figure;
- Non-circular: the criterion of the hierarchy must be respected. In other words, the terms contained in the definition must be already defined.

The concepts of a straight line are generally defined by showing examples as compared to the concept of parallelism which is introduced from a definition. The theoretical framework developed by Vinner (1991), which emphasizes concept image and concept definition, seems relevant to our studies in that the students' explanation of the geometrical objects that we want to study is very often based on their representations. These representations are constructed first from sensitive experience, and then they evolve in a certain abstraction. The question here is to identify through the students' concept definition those that are part of the personal concept definition and evaluating the coherence of their concept image on the studies concepts.

C. Methodology

Our study follows the qualitative research, we have implemented a didactic engineering which is a methodology developed in didactics of mathematics. It is characterized in this study as an experimental design based on the analysis of the students' productions.

1. Population

The population of our study is consisted of 28 students in 8th Grade. They are students aged 13 to 14, all of them were volunteers. French is the first language of our participants and it was in this language that our experiment took place. They have been taught about straight lines and parallel lines in previous classes. It can be assumed that the properties of the concepts of straight lines and parallel line are part of the students' concept image on these concepts. Moreover, in the 8th Grade, students are expected to use their concept image on figures to produce proof. One can assume that the distinction between figure and drawing has been understood by the students at this level. It is assumed, in accordance with Zaslavsky and Shir, (2005), that the student should understand the criteria for a good definition and thus be able to select the essential properties that make it possible to explain a geometric object.

2. Instrument

The instrument of data collection that we have chosen is a test consisting of a question with two modalities. Students are invited to explain the concepts of a straight line and define two parallel lines. Previous work that justifies this approach is that of De Villiers (1998), Fujita and Jones (2007) on students 'comprehension of geometric objects. The fact that students describe a figure allows them to express the elements of their concept image about the figure in question. It allows them to formulate a concept definition to describe their own understanding of the manipulated figure. The data from this experimentation were translated into English. The questionnaire proposed to students is presented as follows.

Question: How can you explain the following figures to someone who doesn't know them?

1. A straight line;

2. Two parallel lines.

Answering the questions posed requires the student to use the properties that he associates with the concepts of a straight line and two parallel lines in their concept image. The student can then produce a concept definition. The experiment took place during the second term of the 2018–2019 school year and was completed in 30 minutes.

3. A priori analysis

A priori analysis of the first modality: explaining what a straight line is.

First of all, it is important to note that to find a formal definition of the straight line is not easy, as this concept is considered by many teachers as a primitive object. However, some textbooks provide acceptable definition for this object (Njomgang & Tchonang, 2018) in this level. Some of these definitions are empirical (which refers to the knowledge of the sensible) and others are picture definitions (the show a picture which represent the figure). Students have studied theory on the straight line and have their concept image on this concept. Based on the properties of the straight line studied in previous classes, students might be expected to propose one of the following definitions:

- Dr. 1: A straight line is an unlimited line on either side; it contains the shortest line between any two of its points.

The answer Dr1 is an answer that we consider acceptable for this grade level. Such a definition may emerge in a student with a certain level of abstraction. The student here uses two properties of the straight line, namely: "to be unlimited on either side" (P1) and "to contain the shortest line between any two of its points" (P2). In fact (P1) reflects the fact that the student's representation of the straight line goes beyond the drawing and is an element of his concept image on a straight line that coincides well with reality. Moreover, the statement of (P2) comes from a reasoning that relies heavily on sensitive experience and refers once again to the student's concept image on a straight line.

Since this definition can be a construction of the student or a definition that was produced in the classroom, it can be seen as a personal concept definition.

- Dr2: a straight line is a set of aligned points

The Dr2 definition comes from the strong link between straight lines and aligned points. Indeed, in order to speak of aligned points, the notion of a straight line must first know. This property of the straight line is relevant; however, it does not satisfy the hierarchy criterion for a good definition. Indeed, someone who does not know the straight line will not be relieved of his ignorance. The person to whom the concept of a straight line is explained should first know what aligned points are. At this level of study, the concept of aligned points is derived from the concept of a straight line. A student who explains the straight line in this way calls up a property of the concept that is not relevant in the situation; the property that is expressed here is part of his concept image evoked. The personal concept definition is not accepted.

- Dr3: A straight line is an unlimited line.

Dr3 is a definition that uses only one property: "being unlimited". This definition is incomplete because it is not sufficient to distinguish the straight line from other unlimited lines. The line here refers to the straight line. This definition is the student's personal concept definition; it allows the student to express his concept image on the straight line. This personal concept definition is not correct.

- Dr4: A straight line is a line.

The evocation of the line, quite simply, refers to the use of the word in the common sense: the line is considered as a straight line. It should be noted that this description can be accepted for a line segment, as long as the students does not specify the unlimited character of the object. Therefore, we can consider that this definition is part of the students' concept image on a straight line and this answer is wrong.

- Dr5: a straight line is a line which is straight

This definition shows that the student makes a categorization of the lines, only the definition does not meet the definition criterion, because the student uses the adjective straight as property to define the concept.

- Dr6: a straight line is a line that has two extremities

The student who produces this definition remains within the empirical geometry: there is indeed a manifestation of the epistemological obstacle due to the limitation of the medium of representation. This is part of the students' concept image.

A priori analysis of the third modality: explanation of two parallel lines

The following answers can be expected:

- Pa1: Two straight lines are parallel when they never touch.

This definition is correct. The student gives the following property to characterize the parallelism between two straight lines: "never touch each other". It takes account the fact that the straight line can be extended indefinitely. A variant of this definition is:

- Pa2: two straight lines are parallel when they do not touch each other.

For a mathematician, this definition can be accepted as a formal definition, because their concept image about the straight line is coherent. But as a precaution, we consider this definition to be incomplete because parallel lines are introduced in empirical geometry, where visual perception plays an important role in the validation of the properties of the figures. The Failure to add the infinite extension on either side could lead to misconceptions about two parallel lines. The answer above may be ambiguous for the reader, as he does not know if the straight line is for the student taken with all his abstraction, or if it is the description of the drawing. This is why we do not consider it to be correct.

- Pa3: two straight lines are parallel when they are perpendicular to the same line.

This definition is an operative definition; it allows both to represent graphically two parallel lines and also intervenes as a warrant in an argument when somebody wants to prove that two straight lines are parallel. It can be accepted as a formal concept definition.

- Pa4: two straight lines are parallel when they, "keep the same distance".

This definition is correct and may reveal a certain level of abstraction on the part of the student who formulates it, just as it may come from observation. In the first case, the students make a paraphrase of a definition sometimes used by teachers: "two parallel lines are equidistant straight lines". It involves the notion of distance between two straight lines. We can infer that this student has a good conception of two parallel lines. In the second case, the student's definition is a description of his mental image of the parallel lines taken from his experience with drawings of two parallel lines. However, the drawing of two parallel lines is partial, which is true on a sheet of paper may no longer be true if the lines were extended indefinitely. The above answer can be considered as a personal concept definition.

Other definitions of two parallel lines may emerge; we will classify them as "other".

D. Findings and Discussion

1. Findings

In this section, we present the students' results of the questionnaire to which they were subjected.

1. The straight line

Table 1: Distribution of answers on the straight line

Dr1	Dr 2	Dr3	Dr4	Dr5	Dr6	others	TOTAL
0	0	E1; E2; E8; E9; E10; E14; E15; E17; E18; E19; E24; E25 **12**	E13 **1**	E16; E21 **2**	E3; E7 **2**	E4; E5; E6; E11; E12; E20; E22; E23; E28; E26; E27 **11**	**28**

The word line is used by 21 of the 28 students to define the straight line. Among them, two students specify that the line is straight. This reflects the fact that when students talk about a line, they are talking about a straight line, a representation that is part of their concept image. Moreover, in school language, the line is very often used to refer to the straight line. The most frequent definition is Dr3 (12/28), which can be explained from our point of view, by the fact that the straight line is not mathematically defined, but always graphical represented with the precision that it can be extended as much as possible. On the one hand, there is a visual experience that is involved in the conceptualization of the straight line, and on the other hand, there is this abstraction that consists of prolonging it, in seeing it as unlimited.

The proposition of E12 caught our attention: *"a straight line is a suite of lines that has no limit or a suite of points that has no limit"*. In this definition, the notion of continuity of the points of the straight is implicit. Moreover, we still find this notion of a line, which refers to the straight line and infinity, which translates as "has no limit". It seems in this definition, a movement towards abstraction. The explanation of the student is connected to the representation of a straight line.

E9 defines the straight line from the straight line: *"a straight line is a straight line that can be extended as far as one wants"*. This incorrect definition shows that the notion of a straight line is intuitive and difficult to express. Moreover, we see that the expression of the unlimited is manifested.

The students who answered Dr6 stay in the visual. In fact, the straight line is represented by a line segment whose extremities are not marked, but which appears clearly on the drawing.

We can conclude that the definitions of a straight line proposed by the students are personal concept definitions. They are often based on sensitive experience, so they are definitions that fall between empirical definitions and mathematical definitions. One explanation for these productions could be the difficulty in describing the property of the straight line that distinguishes it from other lines. For example, an empirical definition of the concept which can be accepted at this level can use the property "being the shortest line between any two of its points". As we mentioned in our a priori analysis, defining the straight line is not easy in elementary geometry. However, some property of the straight line, such as the fact that it is infinite, is part of the students' concept image of the straight line.

2. *Two parallel lines*

Table 3: Distributions of answers on two parallel lines

Pa1	Pa2	Pa3	Pa4	Others	TOTAL
E1 ; E5 ; E17 ; E22 ; E24 ; E26	E2 ; E15 ; E16 ; E18 ; E19 ; E20 ; E21	0	E7 ; E9 ; E12 E3	E4; E6; E8; E10; E11; E13; E14; E23; E25; E27; E28	28
6	7		4	11	

None of the students proposed the answer Pa3. Only four students proposed definitions that refer to the distance between two straight lines. These definitions correspond to the answer Pa4 envisaged in the a priori analysis.

Students E3 and E7 proposed the following answer: "*two parallel lines are straight lines separated by the same distance*". The relationship evoked here is "to be separated by the same distance", which suggests that it has a link with the sensible. This is a definition that does not appear in mathematics textbooks in Cameroon, but students study the link between the notion of parallelism and the distance from a point to a straight line in the 7th Grade. This definition does not satisfy all the criteria of a good definition in mathematics (separated by the same distance seems to be ambiguous). However, it refers to a personal conceptual definition in which an effort of abstraction can be noted. It can be assumed that these students have a coherent concept image on two parallel lines.

A significant number of participants proposed answers that corresponded to the definition Pa2 as expected in the a priori analysis. E 18 proposes the following answer: *"Two parallel lines are two straight lines that do not touch each other."* On the one hand, with regard to what the student proposed as an answer to the first modality, one realizes that he has a coherent concept image of the straight line and consequently his concept image on two parallel straight lines seems coherent. On the other hand, this definition seems to refer to the student's experience with the drawings of two parallel lines and can become an obstacle when it is necessary to recognize the drawing of two parallel lines. In fact, the drawing of two lines is partial on a representation medium, but it is also possible to have the drawing of two intersecting straight lines that do not touch on the same medium.

Unexpected responses emerge from the students' productions and do not correspond to any of the definitions envisaged in the a priori analysis. These descriptions of two parallel lines are not correct because they do not respect the criteria of a good definition in mathematics. The production of student E28 falls into this category: "*Two parallel lines are two straight lines that do not touch each other and have the same length*." This definition is contradictory and related to drawings of two straight lines, because straight lines do not have a length. The student describes his mental image associated with parallel lines, i.e. the drawing of two lines that have the same length. The students' mental images seem to have been constructed from the drawings they have encountered. These may be the drawings that the teacher is accustomed to represent in the classroom, but they may also be the drawings represented in some mathematics textbooks. Most of these drawings have the same shape and orientation and are lines segments with the same length.

Some answers caught our attention, for example those of students E6 and E23, which is as follows: *"They are straight lines that resemble each other."* The property of the concept that emerges from this description is "resemble each other ". This property doesn't have geometrical signification. The student's description the drawing contains in his mental image on two parallel lines. From the point of view of common sense, "resemble each other" could mean having the same form and the same orientation. This definition is a personal concept definition, it is not correct.

Students' productions allow them to express their concept images of two parallel lines. The elements of their concept image evoked here appear to be mental images. They may be the spatial properties of two parallel lines. The drawing has a significant place in students' concept image on two parallel lines. It seems to be responsible for the incoherence observed in the students' concept image on two parallel lines. The difficulties that emerge from the students' productions reflect two types of obstacles: epistemological obstacles due to the way in which students construct their concept image on two parallel straight lines; and didactic obstacles arising from the choices made in textbooks to present this notion.

3. Discussion

In this study, we conducted an experiment with students to find out their conceptions of certain basic geometric figures, namely the straight line and two parallel lines. The results indicate that the students' concept-images of the manipulated figures do not always correspond with the theory.

First of all, our data shows that students have difficulty to find terms to express their concept image evoked on a straight line. The concept image evoked from some students corresponds to: an unlimited line; a line with extremities, etc. In general, students' concept definition indicates that their concept image on the straight line is in conflict with the theory. Students are unable to give the attributes of the line that distinguish it from other lines in geometry. These results are consistent with those of Gutiérrez and Jaime (1999) who studied students' conceptions of the height of a triangle and found that students have a misunderstanding of this concept. The definition of the straight line is not always proposed in textbooks. In some textbooks, the definitions that are proposed are given axiomatically. It is sometimes regarded as a primitive object; however acceptable empirical definitions exist (Njomgang & Tchonang, 2018) and can be constructed by students in the classroom. These results show that students have a misunderstanding of the concept of a straight line; it underlines the pertinence of reconstructing the definitions by the students as suggested by De Villiers (1998). This author considers that the poor mediation of the definition to students by the teacher is a source of difficulty.

Finally, the data from this study show that students have difficulty to describe two parallel lines. The students' answers are personal concept definition which indicates that their concept image is in conflict with the theory of two parallel lines. The students concept images evoked appear to be drawings contained in their mental images. Indeed, the students seem to describe the drawings they have constructed from their experience with the drawings proposed by the teachers and those contained in the textbooks. These visual models are mostly drawings that have the same form and orientation. These phenomena related to drawing correspond to what Fujita and Jones (2007) call prototype phenomena, and this author indicates that these phenomena are the cause of students' difficulties in comprehending the hierarchical classification of parallelograms.

This study found that students have an incoherent concept-image of some fundamental figures in high school geometry due to a poor connection between the components of their concept-image of the figures being manipulated. Moreover, students' definitions were superfluous, ambiguous and contradictory definitions that do not satisfy the criteria of a good definition in mathematics suggested by Zaslavsky and Shir (2005).

E. Conclusion

This study was designed to better understand students' conceptions of certain basic geometric figures (straight line and two parallel lines) through their definition. To achieve this, we opted for a qualitative research, an exploratory study that consists of analyzing students' answers to a questionnaire on the definition of a straight line and two parallel lines. The theoretical framework we have chosen for this study, namely Vinner's concept-image and concept-definition, has proved to be relevant for our analyses.

The results of this study show that students have difficulties in giving characteristics of a straight line and two parallel lines that are conforming to the theory on these geometric figures. The definitions they propose are ambiguous and linked to the drawings of these figures. The productions seem to indicate that they have an incoherent concept image of these figures. Indeed, there seems to be a poor connection between the mental images associated with these figures and their properties in the students' concept-image. Student's difficulties could stem from institutional choices, in particular, the absence of activities of construction of the definition in the textbooks as well as the choices of teachers to illustrate the figures by non-varied drawings.

These results are interesting in that they alert teachers to the choices that are made when introducing a figure in geometry; in particular, the choice of drawings that are used to illustrate the figures and the study of definitions of geometric figures.

This study has limitations. We believe that asking students to draw a straight line or two parallel lines would provide additional information about their concept image. Audio and video recordings of student discussions in problem-solving situations could provide additional information about their concept image on the concepts studied. In addition, the small number of participants does not allow us to generalize our observations. In the light of the results obtained, we can make the following hypotheses:

- The consideration by the mathematical official syllabus in its prescriptions of the activities of construction of the definitions would have a positive impact on students 'comprehension of the study concept. This would put definitions on the same level as theorems.

- If the teacher takes into account the distinction between drawing and figure during the elaboration of the problems destined for the construction of a definition, it will be possible to avoid didactic obstacles. Indeed, a diversity of drawing of figures can prevent prototype phenomenon.

Our future research could consist of understanding in depth how elements of the student's concept image of the figure influence the construction of theorems and the reinvention of definitions of geometric objects. We could also try to understand how the student's conception of the drawing affects the production of arguments in the production of proof.

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