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Supporting the Emergence of Mathematical Knowledge through Problem Posing

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Abstract

Problem posing represents a valuable strategy to create a bridge between mathematics classroom activities and everyday-life experiences. Despite the value of problem posing activities as opportunities for measuring students' mathematical learning outcomes, more research is needed in investigating if and how problem posing could support the introduction of new mathematical knowledge promoting the development of mathematical concepts. The aim of this paper is to start investigating how problem posing can extend students' mathematical knowledge. After having introduced the notion of emergent problem posing, some results from a teaching experiment conducted in a primary school class are reported. The design of the teaching experiment was explicated through the development of the three components of a Hypothetical Learning Trajectory: *learning goal*; *hypothetical learning process*; *learning activities*. Results from the study indicate that semi-structured problem posing activities that start from a suitable artifact could support the emergence of new mathematical knowledge, supporting students' in re-inventing mathematical strategies to solve problems posed by themselves. However, further research is necessary, especially in: supporting the notion of emergent problem posing with more teaching experiments; investigating the role of different artifacts in supporting the process of emergent problem-posing; evaluating which characteristics an artifact should have in order to support the process of emergent problem posing; examining possible relations between students' abilities and emergent problem posing performances.

Keywords: mathematical problem posing, emergent problem posing, hypothetical learning trajectory, realistic mathematics education

A. Introduction

In common teaching the habit of connecting mathematics classroom activities and reality is still substantially delegated to word problems. However, several studies (see Verschaffel et al., 2000; Chairuddin & Farman, 2019; Nasruddin, et al., 2019; Suendarti & Liberna, 2019) have shown that the practice of word problem solving promoted in students the exclusion of realistic considerations. Instead, in order to help students to prepare to cope with situations they have to face out of school, the type of problem solving experiences they are engaged at school need to be rethought (Bonotto, 2013). In particular, realistic and less stereotyped problems that take into consideration the experiential world of students must be inserted in the school practice, in order to create a bridge between mathematics classroom activities and everyday-life experiences. In this direction, problem posing represents a valuable educational strategy that, starting from and working with *rich* and *realistic* contexts (Freudenthal, 1991), can enhance students' reasoning and critical thinking and give sense to their mathematical activity. Indeed, allowing students to write their own mathematical problems may help them to make connections between mathematics in the classroom and their real life (Kopparla et al., 2018), filling the gap between in- and out-of-school mathematical competencies and experiences. If the goal of education is to prepare students for the kinds of thinking they will need, problem posing should be an important part of the curriculum (Singer, Ellerton and Cai, 2015), as requested in many curricular and pedagogical innovation in mathematics education. The main results concerning research on problem posing in mathematics education could be sum up in the following main themes (Ellerton, Singer and Cai, 2015): (i) problem posing can transform attitudes towards mathematics so that the object of mathematics is the problem and not just the solution of a problem; (ii) problem posing can be an agent of change in the mathematics classroom; (iii) through purposeful planning, problem posing can be integrated into school mathematics curricula; (iv) problem posing can be seen as a natural link between formal mathematics instruction, problem solving and modelling. However, when implementing problem-posing activities several difficulties can be encountered (Hansen and Hana 2015), such as:

- *posing mathematically relevant problems*: distinguish between problems that are mathematical relevant and problems that are not is a competence that both students and teachers should become proficient;
- *posing mathematical suitable problems*: which problems are not too difficult neither nontrivial for the students? A fundamental skill is to be able to reformulate problems and choose such contexts that attain a reasonable degree of mathematical sophistication;
- *posing problems such that pupils feel ownership of the problems:* problem posing is an ongoing process, where reformulations and adjustments are required also by students;
- *making problem posing a relevant part of the learning trajectory*: if problem posing should be seen as an integral part of mathematics classes, it must be connected to other mathematical activities in the classroom;
- *incorporating the teaching of mathematical content with problem posing*: two main difficulties can be individuated. The first, not communicating the intent to students and second, posing problems to a little-known mathematical topic, especially when the topic is a specific real-world situation.

Consequently, further research is needed for the future (Ellerton, Singer and Cai 2015), particularly on knowing more about the potential of problem posing to support students' learning. In this direction, the aim of this contribution is to investigate how problem posing can extend students' mathematical knowledge and skills (Klaassen and Doorman, 2015). The starting point will be the introduction of the notion of *emergent problem posing* (see section B.2). Concerning the aim of the study, our hypothesis is that semi-structured (Stoyanova and Ellerton, 1996) problem posing activities that start from suitable artifacts could support the emergence of new mathematical knowledge. To investigate our conjecture, some results from a teaching experiment conducted in a primary school class that evaluated the use of artifacts in fostering emergent problem posing are reported.

B. Literature Review

1. Problem posing

The term problem posing was introduced in education by Paulo Freire in 1970 in his book *Pedagogy of the Oppressed,* as a metaphor for emphasizing critical thinking. Problem posing extended to various domains of knowledge. In mathematics education, problem posing has been identified as an important aspect of mathematics (education) (Christou et al., 2005; Freudenthal, 1973; Ploya, 1954), and more in general as a critically important intellectual activity in scientific investigation.

Since in real life problems must often be created by the solver, the formulation of a problem should be viewed not only as a goal of instruction but also as means of instruction (Killpatrick, 1987; Mashuri, et al., 2019; Djidu, & Jailani, 2017). The advancement of mathematics, in fact, requires creative imagination, which is the result of raising new questions and viewing old questions from a new perspective (Ellerton and Clarkson, 1996). Problem posing, being the act of generating mathematical problems, is a process through which the importance of creativity and critical thinking are emphasized (NCTM 2000). In this perspective, students can actively construct meaning in both the natural and simulated worlds in classrooms. Moreover, teachers and students might create knowledge together in a variety of contexts and generate and address critical questions about the knowledge they produce. In Freire's version, all these aspects could help to develop more democratic, diverse, critically thinking members of society (Singer, Ellerton and Cai, 2015).

Problem posing has been defined by researchers from different perspectives (Silver and Cai, 1996), referring both to the generation of new problems and to the reformulation of given problems (Silver, 1994). In this paper problem posing is considered as the process by which students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems (Stoyanova and Ellerton, 1996). These concrete situations, considered as starting points for the practice of problem posing, could be divided in three categories (Stoyanova and Ellerton, 1996):

1. *free situations*, where students are asked to pose problems without restrictions;
2. *semi-structured situations*, where students are provided with an open situation and are invited to explore its structure and to complete it using their personal previous mathematical experience;
3. *structured situations*, where students pose problems reformulating or varying given problems.

The theoretical arguments supporting the importance of problem posing in school mathematics are supported by a growing body of empirical research. Various aspects of problem-posing had been studied in literature, such as examining thinking processes related to problem posing (Brown and Walter, 1990; Christou et al., 2005), or including problem posing in mathematics activities. In particular, several studies focused on the relations between problem posing and problem solving (Van Harpen and Presmeg, 2013; Cai and Hwang, 2002; Ellerton, 1986) and/or between problem solving and creativity (Xie and Masingila, 2017; Bonotto and Dal Santo, 2015; Bonotto, 2013; Yuan and Sriraman, 2010; Leung, 1997; Silver, 1997, Leung and Silver, 1997; Sriraman, 2009). However, given the value of problem posing activities as opportunities for measuring students' mathematical learning outcomes, there still is a no clear line of research in if and how problem posing activities could support the introduction of new mathematical knowledge promoting the development of mathematical concepts. Consequently, it is mandatory to develop and validate suitable problem posing instruments, understanding which kind of problem posing tasks best reveal students' mathematical understandings (Cai, et al., 2015). In this direction, this paper aims at investigating the role of artifacts in supporting students' emergence of mathematical knowledge.

2. Emergent problem posing

In this section we focus on a particular aspect of problem posing, that we call *emergent problem posing*. To clarify what we mean by emergent problem posing, we start making a connection with emergent modelling.

Emergent modelling was introduced (Gravemeijer, 1999) with the meaning of supporting the emergence of formal mathematical ways of knowing. Indeed, in this perspective modelling activities are used as a vehicle for the development, rather than applications, of mathematical concepts (Greer, et al., 2007). Students, starting from a real context, begin to model their informal mathematical strategies and arrive to re-invent (Freudenthal, 1991) mathematical concepts and applications they need.

As stated in the previous section, problem posing can be defined as the process by which, on the basis of mathematical experience, students formulate meaningful mathematical problems starting from a concrete situation (Stoyanova & Ellerton, 1996). However, often students are not able to solve the problems they pose. In this situation, the problems posed by students that require new mathematical knowledge for their solution can be used as a vehicle to introduce new mathematical concepts. Moreover, these new concepts assume meaning for students, because rooted in their personal experience and for the specific purpose of solving the problems posed by themselves. As a consequence, new mathematical knowledge should emerge from students' posed problems. Similarly to emergent modelling, we call this aspect of problem posing as *emergent problem posing*, highlighting its aim to support the emergence of formal mathematical ways of knowing.

3. Artifacts

In this paper we are investigating if semi-structured problem posing activities that start from a suitable artifact could support the emergence of new mathematical knowledge. As a consequence, only semi-structured problem posing situations will be considered in the rest of the work. Recall that in semi-structured situations students are provided with an open context and are invited to explore its structure and to complete it using their personal previous mathematical experience. Therefore, the choice of such context is fundamental. In this choice we assume the perspective of Realistic Mathematics Education, in which meaningful contexts for mathematical activities are defined as *realistic* and *rich*. A realistic context is given by a situation that is experientially real to students (Gravemeijer & Doorman, 1999). As a consequence, problems should come from the real world, but also from a fantasy world or from the mathematics itself, until they are experientially real for the student. The context must also be rich (Freudenthal, 1991). A rich context is a context that promotes a structuring process as a means of organizing phenomena, physical and mathematical, and even mathematics as a whole, i.e. contexts that give more opportunities in the mathematization process. An example of rich and realistic context is represented by artifacts (Bonotto, 2013). Thanks to its complexity and richness in mathematical meaning, an artifact lives in both the world of symbols and the real one, creating a sort of hybrid space that connects mathematics and everyday contexts. A re-mathematization process is thereby favoured, wherein students are invited to unpack from artifacts the mathematics that has been hidden in them, in contrast with the de-mathematization process in which the need to understand mathematics that becomes embodied in artifacts disappears (Gellert & Jablonka, 2007). As a consequence, movement from common use situations to mathematical structures and vice-versa is allowed. Moreover, by removing some data from an artifact, we can stimulate students to face out with new mathematical goals, such as create new mathematical concepts or applications (Bonotto, 2005). In this direction, we suppose that artifacts could represent a valuable tool to offer students opportunities in emergent problem posing, motivating students in creating new mathematical knowledge from informal contexts.

C. Method

1. Research Design

The claim of this paper is that semi-structured problem posing activities that start from a suitable artifact could support the emergence of new mathematical knowledge. To investigate our conjecture, a teaching experiment in a primary school class was implemented. The teaching experiment was conducted in a fourth-grade class (age 9) of 24 students. The classroom involved in the study had never been engaged in problem posing activities before. The activity was implemented by the author with the presence of the official mathematics teacher. The mathematical topic for the teaching experiment concerned decimal numbers. The teaching experiment covered two mathematics lessons. The design of the teaching experiment was explicated through the development of the three components of a Hypothetical Learning Trajectory (Simon, 1995): *learning goal*; *hypothetical learning process*; *learning activities*.

Students had been introduced to decimal numbers one week before the problem posing activity. In particular, when the intervention took place, students did not know how to perform additions between decimal numbers. Consequently, what we wanted to achieve during the teaching experiment was the re-invention (Freudenthal, 1991) of the algorithm (or more algorithms) to calculate additions between decimal numbers. Therefore, the learning goal of the teaching experiment was represented by addition between decimal numbers.

To design an HLT, together with the learning goal some conjectures about the students' learning process needed to be formulated. Specifically, it was supposed that making students face with a problem situation in which they need a new mathematical concept to solve it could stimulate the same students in creating that concept. The idea consisted in putting students face with a problem posing situation that could stimulate them to pose problems dealing with decimal numbers and that could bring to the need of introducing addition between decimal numbers. Such problem situation should be represented by an artifact (Bootto, 2013), that thanks to its complexity and richness in mathematical meaning, might stimulate students to create new mathematical knowledge from a significant real context. Then, focusing on that problems posed by students which need to develop a strategy to perform addition between decimal numbers, the teacher can foster students' creation of one or more algorithms in a guided re-invention (Freudenthal 1991) way.

Starting from the hypothetical learning process and the learning goal, some learning activities had been designed. The first one consisted in a semi-structured problem posing activity (Stoyanova & Ellerton, 1996), wherein students had to pose problems starting from a given context. One hour was dedicated to this activity. The context chosen for the activity consisted in an artifact represented by a statistic about people practicing sports (Figure 1). In this activity students were asked to pose at least three problems dealing with decimal numbers from the given context. The lesson after the problem posing activity, students were engaged in a problem solving activity, wherein they were asked to solve some problems chosen by the teacher from the ones posed in the previous problem posing activity. In the specific, problems were chosen in order to stimulate students' re-creation of the algorithm of addition between decimal numbers.

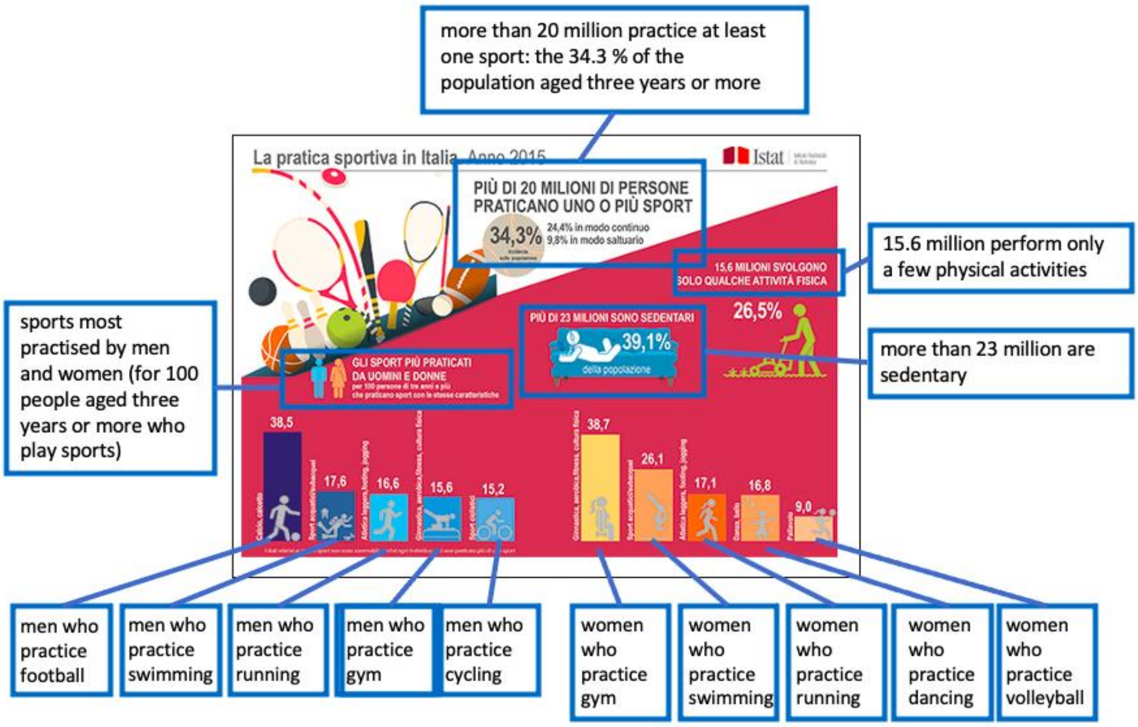

Figure 1: Artifact used for the problem posing activity

2. Data Analysis

For the data analysis, we modified the scheme proposed by Silver and Cai (1996), introducing the new category *emergent problems*. Students' problem posing responses were firstly categorized as *problems* or *statements*. Then, problems were classified as *mathematical* or *non-mathematical problems*. Each mathematical problem was divided between *solvable, emergent* and *not solvable problem*. Problems were considered to be not solvable if they lacked sufficient information or if they posed a goal that was incompatible with the given information. The difference between solvable problems and emergent problems is that for the firsts, students know the mathematics needed to solve them, while the seconds refer to problems that, in order to be solved, require new mathematical concepts. The data analysis scheme is reported in Figure 2. Concerning inter-rater reliability of the scoring, the coding was performed separately by two different researchers. Rates of agreements on the classifications of problems/statements, mathematical problems/non mathematical problems, solvable/emergent/not solvable were highly acceptable: concerning problems/statements, agreement of 98.2% with Cohen's k of 0.82 (almost perfect agreement); concerning mathematical problems/non mathematical problems, agreement of 98.0% with Cohen's k of 0.90 (almost perfect agreement); concerning solvable/emergent/not solvable, agreement of 90.2% with Cohen's k of 0.82 (almost perfect agreement).

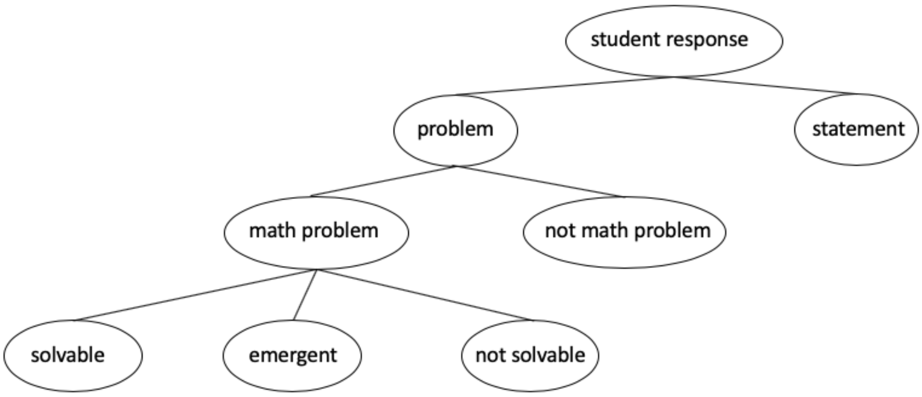

Figure 2: Data analysis scheme

D. Findings and Discussion

1. Findings

In this section results from the described teaching experiment are reported, in order to make some conclusions concerning the initial hypothesis that semi-structured problem posing activities that start from a suitable artifact could support the emergence of new mathematical knowledge.

In Table 1 examples of students' answers for each category of the data analysis scheme are reported.

Table 1: Example of students' answers

Category	Example
Problem	*Football is played by 38.5% of people, while volleyball by 9.0%. How much difference is there?*
Statement	*In one class there are 25 children. 9% practice volleyball, while 15.2% practice cycling.*
Math problem	*In Italy men play various sports: those who play football are 38.5%, while those who swim 17.6%. How much is the total if I add the two percentages together?*
Not math problem	*Maria is swimming. She can go every Friday or Monday. Which day can be more useful?*
Solvable	*There are 300 people practicing sports. 38.5% play football, 17.6% swim. How many people play football? And swimming?*
Emergent	*Male gymnasts are 15.6% and females 38.7%. How many people in percentage practice gymnastics?*
Not solvable	*In my class 26.5% of children play sports, 2.3% do not play sports. How many play more than two sports?*

In Table 2 results of students' problem posing performances are shown. For each category obtained using the data analysis scheme of Figure 2, its numerosity is reported, together with its distribution respect to the total number of students' responses (109). In Figure 3 students' problem posing responses are calculated in terms of *solvable*, *emergent* and *not solvable problems*. In this case, the distributions are calculated respect to the problems classified previously as *mathematical problems*.

Table 2: Students' problem-posing performances

Category	Numerosity	Percentage respect to the total number of students' responses
Problems	102	93,6 %
Statements	7	6,4 %
Math-problems	92	84,4 %
Not math-problems	10	9,2 %
Solvable problems	13	11,9 %
Emergent problems	55	50,5 %
Not solvable problems	24	22,0 %

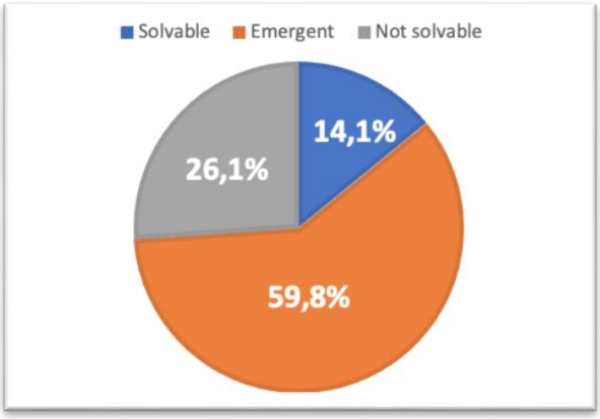

Figure 3: Students' problem posing performances respect to mathematical problems

From Figure 3 it is evident that the majority of the mathematical problems posed by students were actually emergent problems, that means students need to develop new mathematical concepts or strategies to solve such problems. As a consequence, such problems could be used by the teacher in the following problem solving session in order to make students reflect, investigate, develop solving strategies and re-invent mathematical strategies.

Some of the emergent problems posed by students dealt with addition between decimal numbers. In the direction of emergent problem posing, such problems could be used to stimulate and support students' emergence of a mathematical strategy to perform addition between decimal numbers. We report an example of emergent problem, that had been chosen by the author for the next problem solving activity:

P1: *In Italy, women who swim are the 26,1 % and men the 17,6 %. Which is the total percentage of women and men who swim?*

As described in the design of the teaching experiment, in the lesson that followed the problem posing activity, students had to solve some of the problems they posed. In particular, students had to solve such problems in pairs and explain their solving strategies. We present some examples that show different strategies developed by students to solve some emergent problems.

One of the problems was problem P1, that consisted in calculating the total number in percentages of people who practice swimming, i.e. performing 17,6%+26,1%. Some students to perform the calculation, transformed the decimal numbers in fractions and then summed the results: $17{,}6 + 26{,}1 = \frac{176}{10} + \frac{261}{10} = \frac{437}{10} = 43{,}7$, obtaining that the 43,7% is the total percentage of people who practice swimming.

In another case, that consisted in performing 14,5+26,7, students summed separately units and tenths and then summed together the obtained results paying attention to the positional notation (Figure 4).

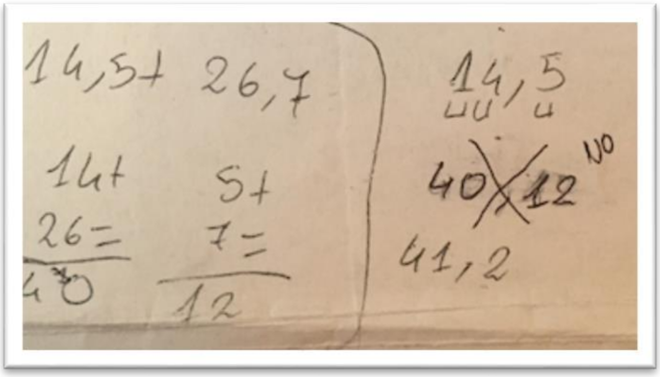

Figure 4: Students' solving strategy to perform addition

Other students were able to reinvent the algorithm for the calculus in column (Figure 5).

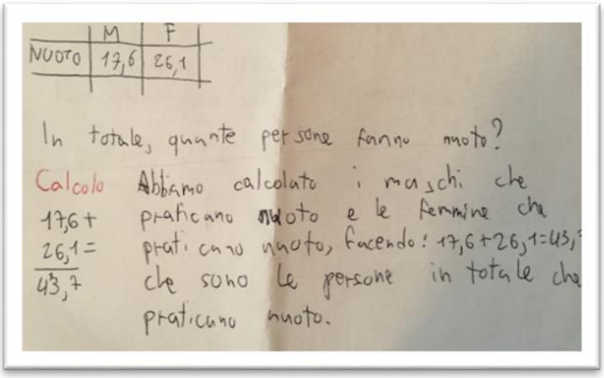

Figure 5: Students' solving strategy to perform addition

2. Discussion

In this section we discuss the results from the teaching experiment in relation to the aim of the study. Recall that our first hypothesis was that emergent problem posing could be supported by the implementation of semi-structured problem posing activities (Stoyanova, & Ellerton, 1996) that start from suitable artifacts.

The students who took part to the teaching experiment did not know how to perform additions between decimal numbers before the activity. During the problem posing session, students were requested to pose some problems dealing with decimal numbers starting from an artifact. Table 2 shows that more than a half (50.5 %) of students' responses had been classified as emergent problems. Such results are in line with Bonotto (2005), wherein the author suggests that artifacts may stimulate students to face out with new mathematical goals, such as create new mathematical concepts or applications. In this study, indeed, the artifact represented by a statistic about people practicing sports (Figure 1), permitted students to pose problems that require new mathematical knowledge to be solved, stimulating students to pose problems concerning addition between decimal numbers. This fact highlights that a rich and realistic context, such as a suitable artifact, could enhance the emergent aspect of problem posing, since students are stimulated to pose problems that require the emergence of mathematical knowledge. However, this process is not finished at this point, instead it is required that such problems created by students should stimulate the same students also in creating a solving strategy through which the needed mathematical knowledge could emerge in practice. In our teaching experiment, students posed several problems concerning addition between decimal numbers. Consequently, to solve such problems, students had been encouraged to re-invent (Freudenthal, 1991) an algorithm to perform such additions. Indeed, starting from their posed problems, the problem solving session that followed permitted students to reflect and produce solving strategies that led to the re-invention of a mathematical strategy to perform addition between decimal numbers. This solving strategy is not limited to the starting situation but can be generalized to perform any addition between decimal numbers. Emergent problem posing, in analogy with emergent modelling, encouraged students in developing mathematical algorithms and procedures starting from their informal mathematical strategies. As a consequence, the problems posed by students supported the same students in creating new mathematical knowledge, fostering in this sense a re-invention (Freudenthal, 1991) process. Moreover, in the activity presented in this paper, students, while solving the problems created by themselves, were able to develop more than one strategy, fact that contrasts the conviction that there is only one possible correct way to solve a mathematical problem (Greer, et al., 2007).

We remark the fact that the kind of context for implementing problem posing activities is fundamental. Indeed, in the teaching experiment presented it was shown that a suitable artifact can foster in a deep and significant way emergent problem posing. Such an artifact was a rich and realistic context that, starting from a meaningful informal situation, supported students in posing mathematical problems that helped them in improving their mathematical knowledge. As a consequence, answering to the aim of the study, our hypothesis was confirmed, that semi-structured problem posing activities that start from suitable artifacts could support the emergence of new mathematical knowledge, expanding in this way students' mathematical knowledge and skills (Klaassen and Doorman, 2015).

E. Conclusion

In this paper we started reflecting on the concept of emergent problem posing. Our claim was that semi-structured problem posing activities that start from a suitable artifact could support the emergence of new mathematical knowledge. To investigate our conjecture, some results from a teaching experiment conducted in a primary school class have been reported. The teaching experiment was divided in two main parts: a problem posing activity and a problem solving one. The problem posing activity was based on a semi-structured situation represented by a rich and realistic context given by an artifact: a statistic about people practicing sports (Figure 1). Starting from such context, the 50.5 % of students' responses had been classified as emergent problems, that actually represented the 59.8 % of the mathematical problems posed by students. Consequently, the problem posing activity, and specifically the artifact used as starting context for the semi-structured (Stoyanova and Ellerton, 1996) problem posing activity, stimulated students in posing problems that for their solution new mathematical knowledge was needed, concerning in particular the development of a mathematical strategy to perform addition between decimal numbers. In this direction, some of students' emergent problems dealing with addition between decimal numbers had been chosen as starting point for the following problem solving session, in order to stimulate the development of a strategy to perform addition between decimal numbers. Findings indicate that the artifact chosen for the problem posing activity enhanced the emergent nature of problem posing, encouraging students in developing mathematical algorithms and procedures to perform addition between decimal numbers starting from their informal mathematical strategies. The problems posed by students supported students in creating new mathematical knowledge, fostering in this sense a re-invention (Freudenthal, 1991) process. As a consequence, our hypothesis is confirmed, proving that semi-structured problem posing activities that start from a suitable artifact could support the emergence of new mathematical knowledge.

This paper represents a starting point in investigating the role of artifacts in fostering emergent problem posing. However, since the research is based on a small-scale sample, the results are not generalizable without further research. Future case studies are needed, in order to validate the scheme proposed for the analysis of students' performance in problem posing and to generalise the results achieved in the current study. Moreover, it is believed that further research is necessary, especially in:

- supporting the notion of emergent problem posing with more teaching experiments;
- investigating the role of different artifacts in supporting the process of emergent problem-posing;
- evaluating which characteristics an artifact should have in order to support the process of emergent problem posing;
- examining possible relations between students' abilities and emergent problem posing performances.

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