Journal of Mathematics Education
~JME~
Website: http://usnsj.com/index.php/JME
Email: editor.jme@usnsj.com

The Study of Geometry in Elementary Mathematics Textbooks in Finland, Singapore, and Taiwan

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ARTICLE INFO

o-ISSN: 2528-2026
p-ISSN: 2528-2468
Vol. 5, No. 1, June 2020
URL: http://doi.org/10.31327/jme.v5i1.1148

Suggestion for the citation and bibliography
Citation in text:
Yang (2020)
Bibliography:
Yang, D. (2020). The study of geometry in elementary mathematics textbooks in Finland, Singapore, and Taiwan. *Journal of Mathematics Education*, *5*(1), 16-29. http://doi.org/10. 31327/jomedu.v5i1.1148

Abstract

This study examined the differences in the presentation of geometry concepts and characteristics of geometry questions among elementary mathematics textbooks in Finland, Singapore, and Taiwan. Horizontal and vertical analysis methods were used to collect and analyze the data. A popular Mathematics textbook from a dominant publisher in each country was selected: Laskutaito from Finland, My Pals are Here! Maths from Singapore, and Kang Hsuan from Taiwan. The results indicated differences in the presentation of geometry concepts among the three textbooks series. In addition, the questions in these textbooks were determined to demonstrate different characteristics. Moreover, these findings highlight the importance of cross-national textbook comparison to enhance the understanding of differences in learning opportunities across different countries.

Keywords: characteristics, elementary school, geometry, mathematics textbook.

A. Introduction

Previous studies have indicated that textbooks play a key role in mathematics teaching and learning (Baker, Knipe, Cummings, Blair, & Gamson,2010; Cai & Ni, 2011; Fan,2013; Reys, Reys, & Rubenstein,2010; Schoen, Ziebarth, Hirsch, & BrckaLorenz,2010; Zhu & Fan,2006). Schmidt et al. (2001) reported a positive relationship between curriculum and mathematics achievement in the Third International Mathematics and Science Study (TIMSS). Törnroos (2005) claimed that learning opportunities was a critical factor influencing students' learning achievement, andstated that nearly 99% of seventh graders in Finland depend on textbooks when learning mathematics. The quality of textbooks influences teaching efficacy and student performance (Fan,2013; Stein, Remillard, & Smith,2007; Tarr, Chavez, Reys, & Reys,2006; Törnroos, 2005). Therefore, conducting a cross-national comparison of mathematics textbooks could be instructive.

Geometry is a major subject in mathematics textbooks and curricula (Finnish National Board of Education,2004; Howson, 1991; Hoyles, Foxman, & Küchemann,2002; Jones & Fujita,2013; Ministry of Education in Taiwan [MEiT], 2008; National Council of Teachers of Mathematics [NCTM],2000; Schmidt et al., 1997). For example, educational organizations such as the NCTM

(2000), MEiT (2008), and Finnish National Board of Education (2004) have all claimed that geometry should be an essential unit in K–12 mathematics textbooks. Atiyah(2001) stated that geometry is one of the "two pillars of mathematics" (p. 657).In addition, studies conducted in recent decades have discussed the type of geometry content that should be included in mathematics textbooks (Jones & Fujita,2013; Sinclair,2008; Usiskin, 1987).

The strengths and weaknesses of mathematics textbooks in specific countries can be determined through international comparative analyses (Cai & Ni, 2011; Fan, 2013; Hiebert et al., 2003; Stigler & Hiebert, 2004). Therefore, this study compared the geometry content of elementary school mathematics textbooks in Finland, Singapore, and Taiwan. The study addressed the following research questions: (1) what are the differences in the structure of the geometry materials presented to first through sixth graders among the three textbooks series? and (2) what are the characteristics of the geometry questions in the three textbooks series?

B. Literature Review

1. The Importance of Studies on Mathematics Textbooks

Considerable research has indicated that the quality and editing process of textbooks directly influence teaching quality and students' learning achievement (Fan, Zhu, & Miao, 2013; Reys & Reys, 2006; Stein, Remillard, & Smith, 2007; Tar et al., 2008; Törnroos, 2005). Park and Leung (2006) indicated that many East Asian teachers and students regarded textbooks as "bible[s]." Numerous studies have claimed that in addition to textbooks being the main learning tool guiding teachers in elementary through junior high school classrooms, they are the main learning resource for students (Cai, Nie, & Moyer, 2010; Fan, 2013; Huang & Cai, 2011; Reys & Reys, 2004). Thus, mathematics textbooks play a crucial role in school mathematics curricula (Cai & Ni, 2011; Fan et al., 2013; Provenzo, Shaver, & Bello, 2010). This has encouraged many mathematics educators to conduct textbook-related studies (Baker et al., 2010; Cai, Wang, Moyer, Wang, & Nie, 2011; Fan, 2013; Reys et al., 2010; Schoen et al., 2010; Usiskin & Willmore, 2008) and international comparative analyses of mathematics textbooks (Yang, Reys, & Wu, 2010; Cai & Ni, 2011; Charalambous, Delaney, Hsu, & Mesa, 2010; Ding & Li, 2010).

Research has indicated that such international comparative analyses can be used to conduct more in-depth investigations of the effects of textbooks on teachers' teaching and students' learning performance (Stein et al., 2007; Tarr et al., 2006; Tar et al., 2008; Törnroos, 2005). Furthermore, international comparative analyses can determine not only the differences in the arrangement and design of textbooks from various countries but also their strengths and weaknesses. The results can serve as a reference for education authorities, curriculum designers, and publishers in editing or creating new textbooks (Cai & Ni, 2011; Fan et al., 2013; Hiebert et al., 2003; Stein et al., 2007; Stigler & Hiebert, 2004).

2. Geometry Topics in Studies on Elementary Mathematics Textbooks

As discussed, many studies have compared and analyzed mathematics textbooks and determined the roles that textbooks play in mathematics classrooms worldwide (Yang et al., 2010; Cai & Ni, 2011; Cai et al., 2011; Charalambous et al., 2010; Fan, 2013; Fan et al., 2013; Jones & Fujita, 2013; Zhu & Fan, 2006). Specifically, several studies have examined differences in the content, structure, and presentation of fractions, numbers and operations, and early algebra in elementary school mathematics textbooks (e.g., Yanget al., 2010; Alajmi, 2012; Charalambous et al., 2010; Ding & Li, 2010; Son & Senk, 2010). In addition, numerous studies have examined differences in the geometry (Hoyles et al., 2002; Jones & Fujita, 2013; Peterson, 2008) and reasoning-and-proving lessons presented in secondary school (Cai & Cirillo, 2014; Fujita & Jones, 2014; Miyakawa, 2012; Thompson, 2014) and elementary school (Bieda, Ji, Drwencke, & Picard, 2014) textbooks. However, an international comparative analysis has not yet been conducted to determine the topics or characteristics of geometry units in elementary school mathematics textbooks. Therefore, the current study conducted such an analysis of textbooks from Finland, Singapore, and Taiwan.

3. Characteristics of Geometry Questions in Studies on Mathematics Textbooks

The characteristics of the geometry questions were defined as the uniqueness or difference in the sequence of learning materials among the three textbooks. Uniqueness implies the presence of special content in one country's curriculum. For example, Finnish textbooks contain a uniquely

diverse range of questions on visual fractions (Yang, 2018) and use a wide variety of pictures, as compared with Taiwanese textbooks, to help children develop fraction concepts (Yang, 2018). The difference in the sequence of learning materials refers to the order in which certain content is presented. For example, in Taiwan, the concepts of "perpendicular" and "parallel" are introduced in second grade through examples from everyday life, followed by formal definitions in fourth grade. However, in Singapore, direct definitions of these terms are provided in third grade, and in fourth grade, these definitions are applied to everyday situations (Yang, 2018).

Previous studies have argued that the organization of mathematics content not only plays an important role in mathematics curricula but also affects students' learning opportunities (Fan et al., 2013; Cai et al., 2011; Grouws Tarr, Chávez, Sears, Soria, & Taylan, 2013; Tarr, Grouws, Chávez, & Soria, 2013). Furthermore, Tarr et al. (2013) indicated that learning opportunities could be used to predict students' learning outcomes. Studies have shown that the styles in which mathematics questions are presented in textbooks directly affect students' learning opportunities (Cai & Ni, 2011; Fan et al., 2013; Stein et al., 2007) and that the types and approaches of questions posed in these textbooks affect the teaching efficacy of teachers and the learning outcomes of students (Fan et al., 2013; Reys, Reys, Chavez, 2004; Tarr et al., 2013). Moreover, Cai (1995) showed that U.S. textbooks tend to use visual representations to solve mathematics questions more often than their Chinese counterparts. According to the study, 13% of U.S. students preferred to use visual representation methods to solve mathematics questions; however, no Chinese students indicated such a preference. Zhu and Fan (2006) believed that such outcomes are due to mathematics textbooks in the U.S., placing comparatively greater emphasis on visual representation questions. Therefore, the characteristics and differences in mathematics textbooks arguably affect students' learning opportunities. The advantages and disadvantages of the mathematics textbooks in a particular country can be determined through cross-national textbook comparisons (Yang et al., 2010, 2018; Hiebert et al., 2003; Stigler & Hiebert, 2004).

C. Method

1. Selection of Mathematics Textbooks

Kang Hsuan in Taiwan

Kang Hsuan (K.H.) elementary mathematics textbooks are the most widely used in Taiwan, with an approximately 38% market share (Kang Hsuan Educational Publishing Group, 2010). The editing objectives of K.H. are to connect mathematics to real-life situations and improve communication and problem-solving abilities. The K.H. series comprises 12 student textbooks for grades one through six. Each textbook contains eight or nine units, for a total of 115 units in the series. Twenty-one units include geometry topics.

Laskutaito in Finland

The Laskutaito set of mathematics textbooks are published by the Werner Söderström Corporation (Rikala, Sintonen, Uus-Leponiemi, Ilmavirta, & Sieppe, 2006) and are based on the core mathematics curriculum developed in Finland in 2004. The teaching objectives of these textbooks are to assist students in developing profound mathematical understanding and cultivate thinking, reasoning, problem-solving, and creative abilities (Rikala et al., 2006). The Laskutaito series accounts for the largest market share (approximately 70%) of elementary school mathematics textbooks in the country (WSOY, 2006). The Laskutaito comprises 54 units for first through sixth grades, with six units dedicated to geometry.

My Pals are Here! Maths in Singapore

My Pals are Here! Maths (MPHM) is a set of mathematics textbooks edited by Fong, Ramakrishnan, and Gan (2005). MPHM has an approximately 70% market share of elementary school mathematics textbooks nationwide (Yanget al., 2010). The teaching objectives of MPHM entail assisting students in developing basic mathematics concepts, creativity, and critical thinking ability, in addition to problem-solving skills (Ministry of Education in Singapore, 2001). MPHM comprises 93 units for first through sixth grades, with 21 units dedicated to geometry

2. Analysis of Analytical Framework

Horizontal and vertical analyses were used in this study on the basis of previous studies (Charalambous et al., 2010; Hong & Choi, 2014). Charalambous et al. (2010) suggested that

horizontal analysis can provide background information and clarify the overall structure of textbooks, including the number of lessons and questions and the specific geometry topics. In addition, vertical analysis supports an in-depth understanding of the characteristics of geometry content.

In the present study, the horizontal analysis was used to determine the number of geometry lessons and questions included in each textbook series, and vertical analysis provided an in-depth understanding of the characteristics of the geometry questions. A framework and a coding scheme were developed to compare the presentation of the geometry content in the three textbook series. The analysis focused on two aspects: the structure of the geometry materials and the characteristics of the geometry questions.

3. Structure of the Geometry Materials

Analyzing the structure of the geometry materials entailed counting the number of geometry questions, chapters, and lessons. The questions and exercises presented in the student textbooks were counted to determine the total number of geometry questions. For example, because there were four answer blanks in the exercise "Who sees the object from this viewpoint?" (see Figure 1), it was counted to contain four questions.

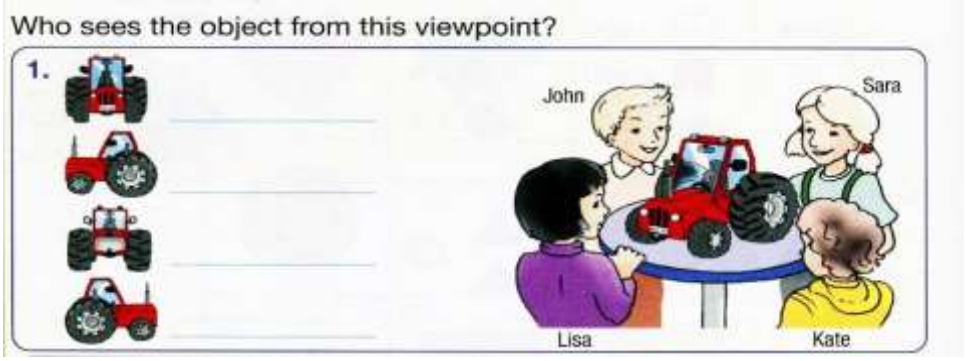

Figure 1. Example of counting questions and visual form from Laskutaito textbook Note. From Laskutaito 2B (Rikala et al., 2006, p. 44)

For all grade levels of each textbook series, all chapters and topics with "geometry" in the title were analyzed. The table of contents of each textbook was checked to determine whether any chapters are dedicated to geometry (Alajmi, 2012; Ginsburg, Leinwand, Anstrom, & Pollock, 2005; Jones & Fujita, 2013). As shown in Table 1, two chapters in the Taiwanese textbooks (i.e., "Square Boxes, Round Cans, and Spheres" and "Making Shapes") and one chapter each from the Finnish ("Geometry and Measuring") and Singaporean ("Shapes and Patterns") textbooks are dedicated to geometry.

In addition, each topic in these selected chapters having titles referring to geometry, defining geometry concepts, discussing examples, or providing practice exercises was counted as one lesson (Alajmi, 2012). As presented in Table 1, four topics in the "Shapes and Patterns" chapter of the Singaporean first-grade textbooks ("Getting to Know Shapes," "Making Pictures from Shapes," "Seeing Shapes in Things Around Us," and "Getting to Know Patterns") were counted as lessons. Six lessons were counted in the Taiwanese first-grade textbooks and one lesson in the Finnish first-grade textbooks.

4. Characteristics of the Geometry Questions

The characteristics were defined as the uniqueness or difference in the sequence of learning materials. Uniqueness refers to the presence of specific content in only one country. For example, mathematical symbols such as $\perp$ (perpendicular) and // (parallel) are applied in MPHM but not in K.H. or Laskutaito (each of which applies only the words "perpendicular" and "parallel"). The difference in the sequence of learning materials refers to certain learning content being presented in differing order (e.g., the previously described differences in how the concepts of "perpendicular" and "parallel" were introduced).

Table 1. Geometry chapters and topics for the first grade covered in the textbooks

Taiwan KH	Finland Laskutaito	Singapore MPHM
1. Square box, Round cans, sphere	**1. Geometry and measuring**	**1. Shapes and Patterns**
(1). getting to know the three-dimensional shape	(1) distinguish shapes	(1) getting to know shapes
(2) getting to know the two-dimensional shape		(2) making picture from shapes
(3) counting the number of different shapes		(3) seeing shapes in things around us
2. Making shapes		(4) getting to know patterns
(1) copying shapes		
(2) lying flat		
(3) three-dimensional stack		

5. Reliability and Validity

The author and two assistants served as raters to check the coding reliability. On the basis of the coding framework, the raters independently coded the questions in the student textbooks. This study applied the measures recommended by Wang (1996). The average mutual agreements among the raters on the student textbooks were 0.835, 0.826, and 0.852; the final reliability was 0.939.

D. Findings

1. Structure of the Geometry Materials

Table 2 presents by grade the total numbers of chapters, lessons, and questions in each series and the total numbers of chapters, lessons, and questions specifically related to geometry.

Table 2. Number of chapters, lessons, and questions containing geometry in the textbooks

Country	Grade	# of chapters in each book	# of chapters on geometry	# of lessons in each book	# of lessons on geometry	# of questions in each book	# of questions on geometry
Finland	1	10	1	34	4	2635	104
	2	8	1	39	5	2621	168
	3	10	1	63	4	2910	133
	4	10	1	63	5	3140	204
	5	8	1	76	14	2927	307
	6	8	1	72	12	2584	183
	Total	54	6(11.1%)	347	41(11.8%)	17030	1099(6.5%)
Singapore	1	19	1	66	4	963	51
	2	17	2	71	5	1062	78
	3	18	3	63	13	1115	113
	4	14	6	54	17	1294	298
	5	14	5	61	14	2007	425
	6	11	4	25	6	1204	391
	Total	93	21(22.6%)	340	59(17.3%)	7645	1356(17.7%)
Taiwan	1	18	2	69	6	627	69
	2	18	2	70	7	887	78
	3	20	3	78	11	995	133
	4	22	5	84	20	1160	287
	5	18	5	72	18	1012	166
	6	20	6	72	18	953	196
	Total	116	23(19.8%)	445	80(18.0%)	5634	929(16.5%)

The data revealed the design of these textbook series to differ by country. For example, the Finnish textbook series has a total of only 8–10 chapters and only one chapter per grade containing geometry; however, the Singaporean textbook series has a total of 11–19 chapters and 1–6 chapters per grade containing geometry. The Taiwanese textbook series has a total of 18–22 chapters and 2–6 chapters per grade dedicated to geometry. The Taiwanese textbook series has the most lessons and highest percentage (18.0%) devoted to geometry, whereas the Finnish textbook series has the fewest lessons and lowest percentage (11.8%). The Singaporean textbooks have the most geometry questions and the highest percentage of questions dedicated to geometry (17.7%) for the first through sixth grades; their Finnish counterparts have the lowest percentage

(6.5%) for those grades. In Finland, the fifth-grade textbook places the greatest emphasis on geometry, with 14 lessons and 307 questions related to geometry. In Taiwan, by contrast, the fourth-grade textbook has the greatest emphasis, with 20 lessons and 287 questions related to geometry. In Singapore, the fourth-grade textbook has the most lessons related to geometry, and the fifth-grade textbook includes the most questions.

2. Characteristics of the Geometry Questions

Finnish Textbook Series

First, the geometry questions for each grade in the Laskutaito mathematics textbooks are closely connected to real-life situations. For example, Textbook 2B contains the exercise "Who sees the object from this viewpoint?" (see Figure 2). This type of question assists children in perceiving the usefulness of mathematics in daily life.

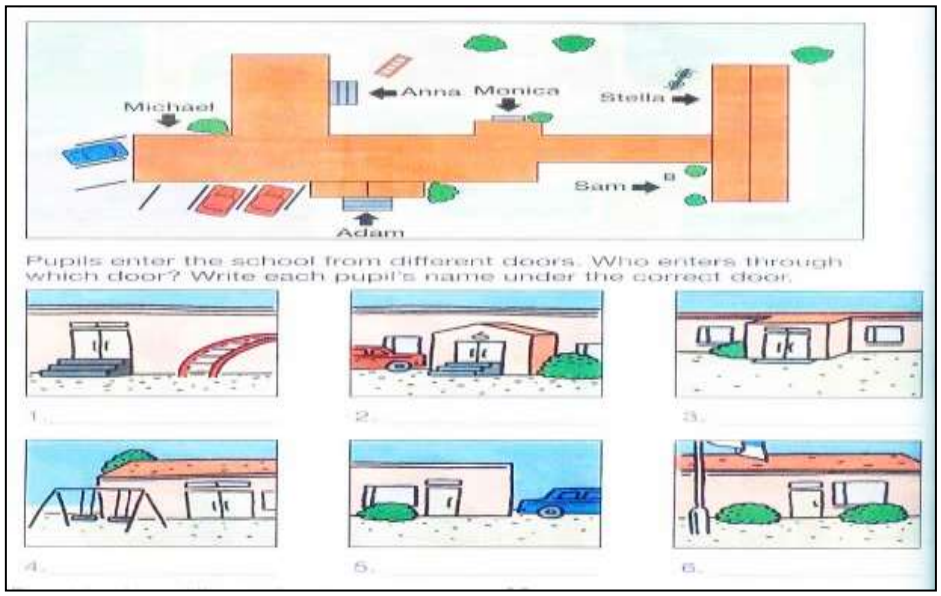

Figure 2. Example from Laskutaito textbook
Note. From Laskutaito 2B (Rikala et al., 2006, p. 44)

In addition, Textbook 3A presents the following exercise: "Pupils enter school from different doors. Who enters through which door? Write each pupil's name under the correct door" (see Figure 3). Learning to read a map and locate the correct door connects mathematics to frequent map usage in school environments.

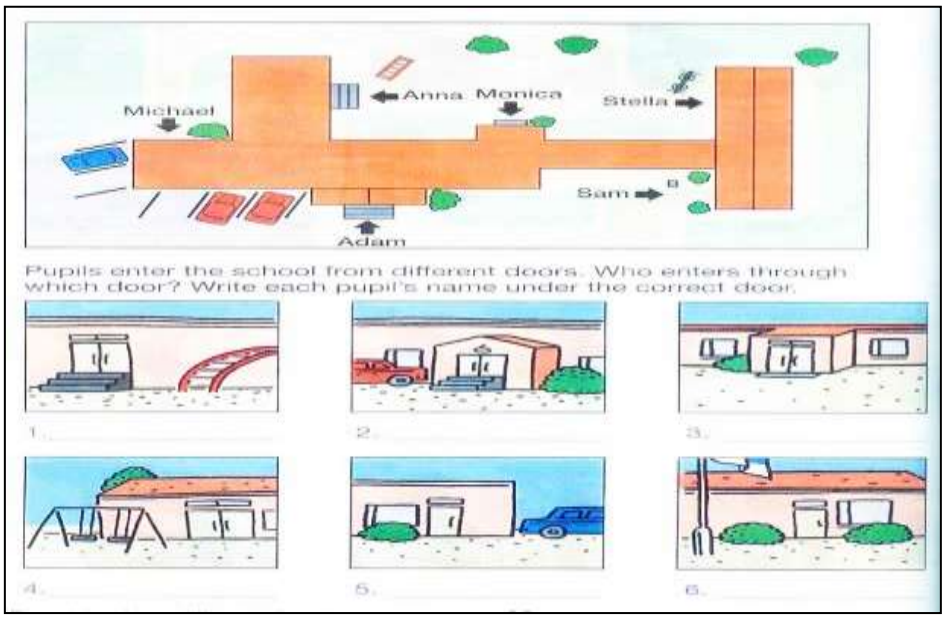

Figure 3. Example from Laskutaito textbook
Note. From Laskutaito 3A (Rikala et al., 2006, p. 88)

In a further example of questions related to concrete, everyday experiences, Textbook 4A includes the exercise “From which form of transport (A–F) does the scenery look like this?” (see Figure 4). These questions were collectively coded as a unique characteristic of Finnish textbooks as compared with Taiwanese and Singaporean textbooks.

Figure 4. Example from Laskutaito textbooks
Note. From Laskutaito 4A (Rikala et al., 2006, p. 82)

Second, the Laskutaito mathematics textbooks place comparatively greater emphasis on learning by doing and practical questions. Figure 5 shows geometry questions for first through fourth grades that involve drawing graphs by hand; each question requires children to draw a graph according to the guidelines provided. This type of question promotes children's ability to integrate themselves into problem-solving activities and encourages them to solve questions through learning by doing. Therefore, this style of the question was coded as another unique characteristic of Finnish textbooks.

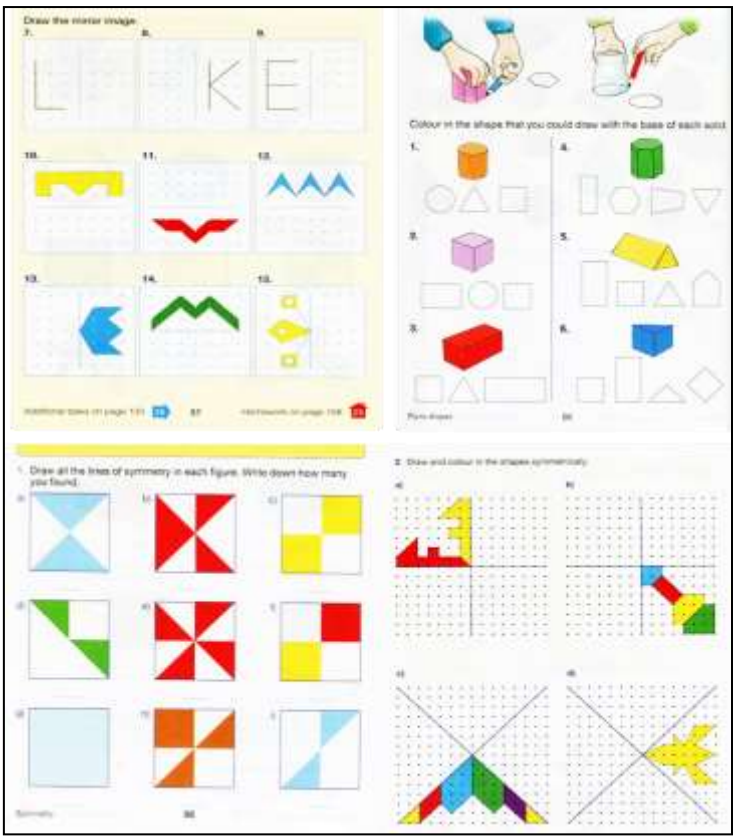

Figure 5. Examples from Laskutaito textbooks
Note. From Laskutaito 1B, 2B, 3A, 4A (Rikala et al., 2006, pp. 66, 57, 98, 84)

Third, many complex graphs are presented in the Laskutaito mathematics textbooks to assist first through fourth graders in developing their understanding of geometry concepts (Figure 6). Such graphs are not used in the Taiwanese or Singaporean textbooks. For example, Graphs 10, 11, and 13–16 (p. 50) and Graphs 13–15 (p. 59) in Textbook 2B are highly complex for the second-grade level. Such graphs would present a considerable challenge to Taiwanese second graders. These questions strongly encourage children to perform mathematical thinking and increase their spatial sense. Therefore, this graph usage was likewise coded as a unique characteristic of Finnish textbooks.

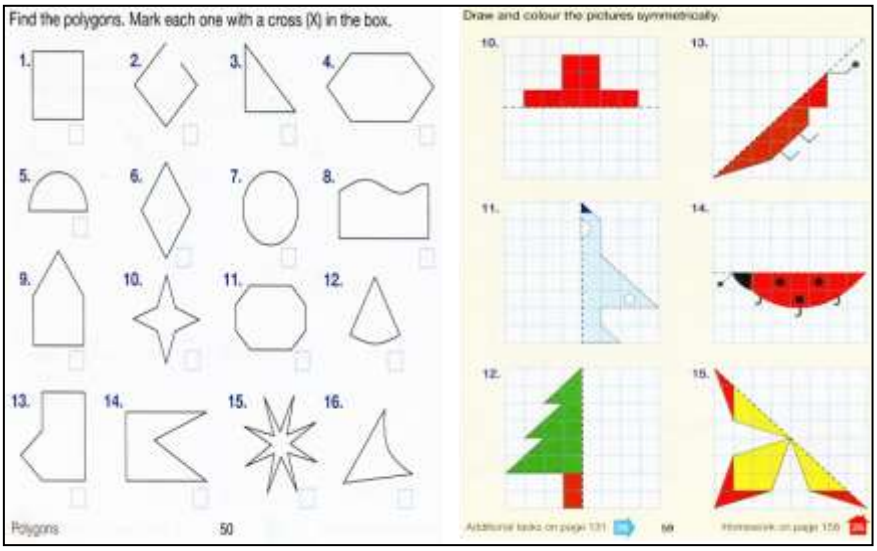

Figure 6. Example from Laskutaito textbooks
Note. From Laskutaito 2B (Rikala et al., 2006, pp. 50, 59)

Singaporean Textbook Series

First, MPHM uses symbolic, visual, and verbal representations to introduce geometry concepts in questions, such as the definitions of a square and a rectangle and the sum of angles in a triangle (Figure 7). In particular, highlighting a visual representation while providing other representations can assist children in easily perceiving geometry concepts. Therefore, this was coded as a unique characteristic of Singaporean textbooks as compared with Taiwanese and Finnish textbooks.

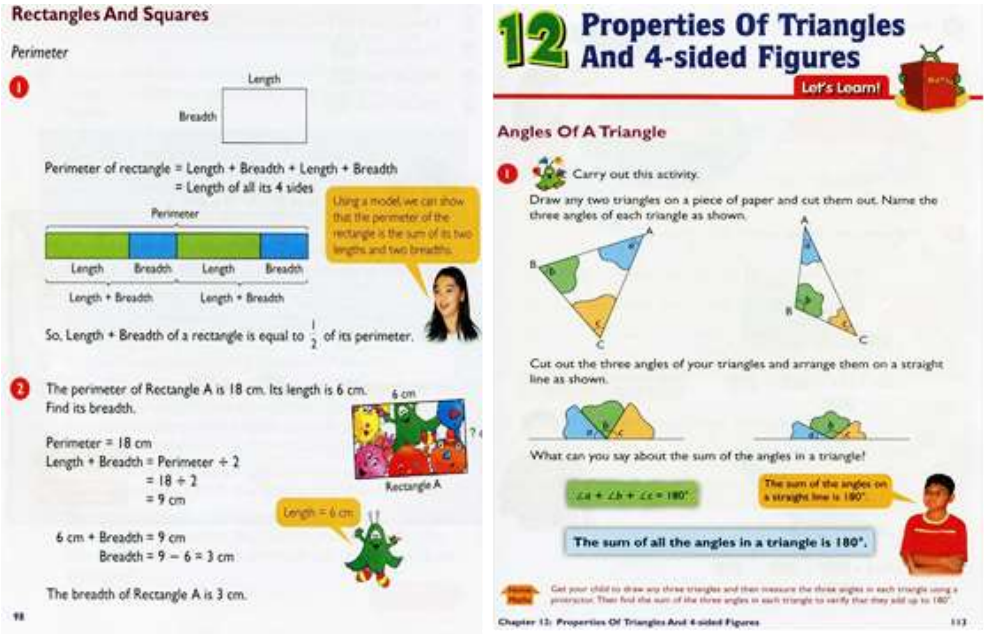

Figure 7. Example from MPHM textbook
Note. From MPHM 4B (Fong, Ramakrishnan, and Gan, 2005, p. 98)

Second, mathematical symbols such as perpendicular (⊥) and parallel (//) are used in MPHM but not in K.H. textbooks (which use only the words "perpendicular" and "parallel" in Chinese). Using these symbols at the elementary school level assists students in transitioning to mathematics learning at the middle school level. Therefore, this was coded as another unique characteristic of Singaporean textbooks.

Third, this study observed an obvious difference in the sequence for teaching perpendicular and parallel lines. In Taiwan, the K.H. textbooks first introduce the concepts of horizontal and vertical lines in real-life situations in the second grade. However, the formal definitions for perpendicular and parallel lines are not provided until the fourth grade. This teaching sequence differs from that used in the MPHM textbooks, which first introduce the definitions of perpendicular and parallel lines in the third grade and apply horizontal and vertical lines in real-life situations in the fourth grade. Therefore, this was coded as another unique characteristic of Singaporean textbooks due to the difference in the sequence of learning materials as compared with Taiwanese or Finnish textbooks.

Fourth, “Geometrical Construction,” a special chapter in the MPHM fifth-grade textbook, includes two lessons (i.e., “Drawing Triangles” and “Drawing Four-sided Figures”) not found in Laskutaito or KH textbooks. These lessons instruct students on how to draw triangles and four-sided figures (e.g., squares, rectangles, rhombuses, and parallelograms) by using different tools such as a ruler, a protractor, or a set-square. Both lessons can help children to develop a deep understanding of the crucial topic of geometric construction. Therefore, this was coded as another unique characteristic of Singaporean textbooks.

Taiwanese Textbook Series

Stories drawn from history are presented on the first page of each chapter on geometry in K.H. textbooks. These thematically relevant stories can help children make solid connections between geometry and mathematical history. For example, at the beginning of Chapter 3 ("Cylinders and Pyramids") in the sixth-grade textbook, a historical account is used to introduce cylinders and pyramids (Figure 8). Chapter 7 ("Area of a Circle") of the same textbook begins with a story of how the area of a circle was calculated by a Chinese mathematician approximately 1,700 years ago in the first Chinese mathematics book, Nine Chapters on the Mathematical Art (Figure 9).

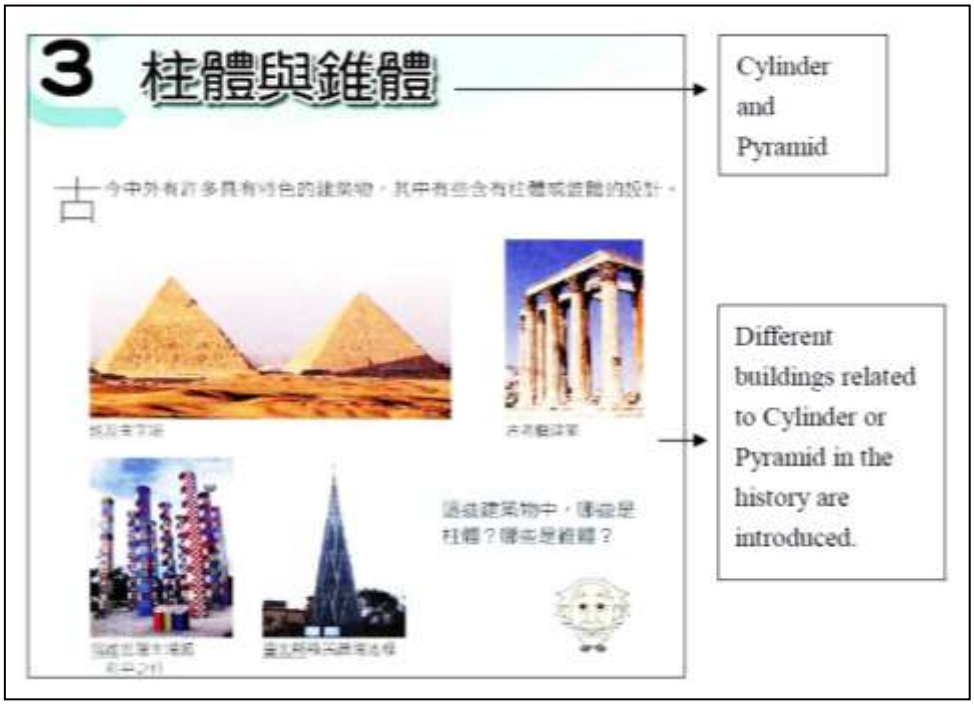

Figure 8. Example from K.H. textbook
Note. From KH 6A (Kang Hsuan Educational Publishing Group, 2010, p. 37)

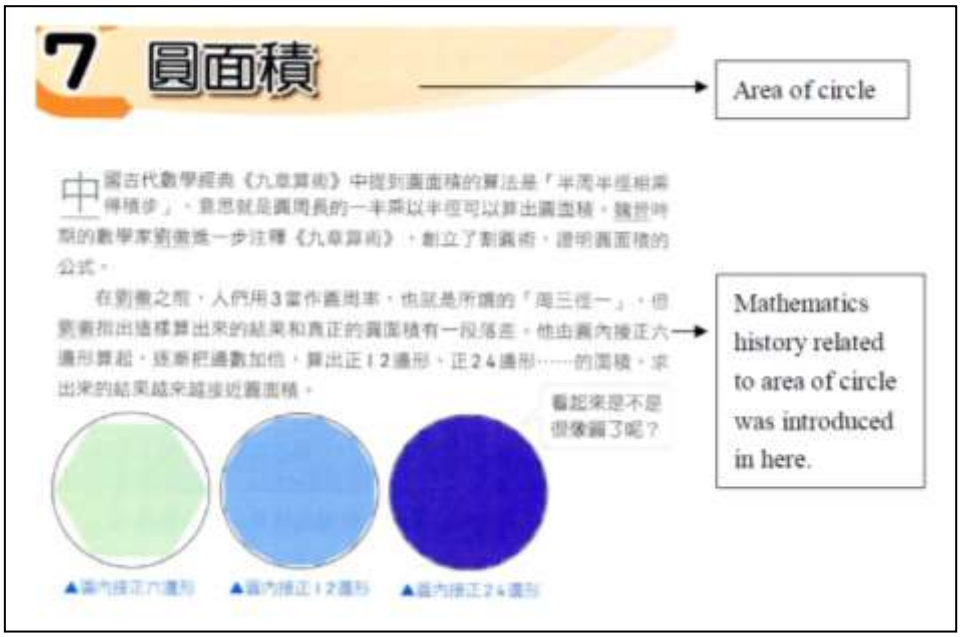

Figure 9. Example from K.H. textbook
Note. From KH 6A (Kang Hsuan Educational Publishing Group, 2010, p. 91**)**

E. Discussion and Conclusion

This study compared the mathematics textbook series used in three countries: Finland, Singapore, and Taiwan. The analysis focused on differences in the structure of the geometry materials and the characteristics of the geometry questions. The results reveal that the structure of the geometry materials differs among the three mathematics textbook series. This study determined the Singaporean textbook series to include the highest percentages of chapters and questions related to geometry and the most geometry questions for all studied grade levels. The study also determined the Finnish textbook series to include the lowest percentages of chapters, lessons, and questions related to geometry. This indicates that the Singaporean textbooks provide students with more learning opportunities and materials related to geometry. Previous studies have suggested that textbooks exert a key influence on students' learning opportunities (Cai, Wang, Moyer, Wang, & Nie, 2011; Fan, 2013; Fan et al., 2013; Haggarty & Pepin, 2002), which crucially affects students' mathematics achievement (Törnroos, 2005). The rich offering of geometry questions in Singaporean textbooks may substantially contribute to the top marks Singaporean students continually receive in international mathematics rankings such as the TIMSS and Programme for International Student Assessment.

The three countries were also shown to present geometry questions differently. This supports earlier studies that have suggested that mathematics textbooks differ by culture (Yang et al., 2010; Cai & Ni, 2011; Charalambous et al., 2010; Ding & Li, 2010; Fan, 2013). The present study determined several specific characteristics of the *Laskutaito* and *MPHM* series. The geometry questions in the *Laskutaito* textbooks are closely connected to concrete situations, enabling them to clearly portray the usefulness of mathematics in children's everyday lives. These textbooks place comparatively greater emphasis on learning by doing, thus assisting children in developing their understanding of geometry concepts through practical activities. In addition, numerous complex graphs are used to assist first through sixth graders in developing their understanding of geometry concepts. These graphs are not used in the Taiwanese or Singaporean mathematics textbooks. Moreover, the geometry questions in the *MPHM* textbooks use multiple representations to introduce geometry concepts. This correlates with the findings of previous studies that the use of multiple representations promotes students' mathematical learning and understanding (Yang & Huang, 2004; Cai, 1995).

Geometry is a critical topic in mathematics curricula (Finnish National Board of Education, 2004; Howson, 1991; Hoyles et al., 2002; Jones & Fujita, 2013; MEiT, 2008; NCTM, 2000; Schmidt et al., 1997) and plays a crucial role in daily life and advanced mathematics learning (Jones & Fujita, 2013; Sinclair, 2008; Usiskin, 1987). Students should be given opportunities to observe geometry in their daily lives and appreciate the importance of comprehending geometry-related topics. The design of geometry content in elementary school mathematics textbooks should, therefore, focus

on using multiple representations, connecting geometry with everyday situations, and teaching mathematics through hands-on activities (NCTM, 2000).

Implications and Limitations

This study focused on examining the presentation of geometry concepts and characteristics of the geometry questions in three series of elementary mathematics textbooks from three different countries. Although the generalizability of the present study findings may be limited, some key implications are provided for mathematical textbook developers, teachers, and researchers. First, this cross-national comparison calls attention to differences in the presentation of geometry concepts among the three countries. For example, the Singaporean textbooks provide more learning opportunities for geometry questions than the other two series. Second, the differing characteristics of geometry questions among the three countries demonstrate the influence of cultural diversity.

The advantages of geometry questions in textbooks from different countries can serve as examples for future revisions of textbooks, design of textbooks, or the selection of useful examples from other countries for integration into mathematics classrooms. For example, the specific characteristics of the *MPHM* series involve the use of multiple representations to introduce geometry concepts and the special topic of "Geometrical Construction" to extend students' geometry proofs and reasoning through the application of various tools. The distinguishing features of geometry questions in the *Laskutaito* textbooks include a close connection to real-life situations and emphasis on learning by doing. These advantages of Singaporean and Finnish textbooks can be considered by Taiwanese textbooks designers, teachers, and researchers for future textbook revision or design. Future studies could explore how these different textbook features affect students' learning and "the extent to which teachers can capitalize on the affordances of the textbooks they use or, alternatively, help overcome textbook limitations" (Charalambous, 2010, p. 147). Considering the limitation that only one textbook series was examined for each country and that more than 70% of students in Finland and Singapore and approximately 40% of students in Taiwan use the textbooks we examined (i.e., numerous students use other textbook series), caution should be applied when generalizing or otherwise applying the results of this study.

F. Acknowledgement

This article was a part of a research project supported by the Ministry of Science and Technology, Taiwan, with grant no. MOST 105-2511-S-415-003-MY3. Any opinions expressed here are those of the authors and do not necessarily reflect the views of the Ministry of Science and Technology.

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