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Deductive Reasoning of Student Teacher Candidates: A Study of Number Theory

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Abstract

Deductive reasoning which includes generalizing, justifying, exemplifying, comparing, and classifying is the main feature of studying mathematics. This study aims to describe qualitatively the deductive reasoning of second-semester mathematics teacher candidates in studying number theory. This research is a qualitative descriptive study with mathematics teacher candidates who have equal mathematical abilities and are of the same sex, namely women as research subjects. The results showed that the two subjects met the indicators of deductive reasoning, namely making general statements, making special statements, and concluding. This could be caused by the characteristics of prospective teacher students in receiving, storing, processing, and how to solve problems or what is called cognitive style.

Keywords: *Deductive Reasoning, Student Teacher Candidates, Number Theory*

A. Introduction

(Loewenberg Ball et al., 2008), understanding mathematics has no meaning without serious reasoning emphasis. When an individual has an understanding without reasoning, the understanding that the individual has is meaningless. Reasoning and proof are the basis of mathematical understanding. Learning to think and reason is very important for the growth of mathematical knowledge. In the process of justifying, individuals naturally build their arguments when doing proof or solving problems. Reasoning and proof form the foundation of understanding mathematics.

(Mueller & Maher, 2009), reasoning and proof are the basis of mathematical understating The study of proof in matters is often associated with deductive reasoning which requires analytical reasoning. Deductive reasoning which is logical reasoning is a pillar of reasoning related to mathematical proof and argumentation, even related to communication and problem-solving. Difficulties in using deductive reasoning are experienced by mathematics students at various levels of education. Likewise, difficulties in deductive reasoning are experienced by student teacher candidates, especially math teacher candidates. Prospective mathematics teachers usually experience many obstacles and even fail to carry out formal proof. This is also usually experienced by most people when faced with a problem situation that requires logic in solving it.

(Hegel, 2001), all reasoning is thinking, but not all thinking is reasoning. Only the thought process that is based on data, evidence, or systematic rationale for concluding is reasoning. There are many mental processes or types of thinking that differ from reasoning. Someone can remember or imagine something without reasoning, or it can be said that someone thinks not necessarily reasoning.

Deductive reasoning needs to be developed in problem-solving or when someone is faced with conditions that are not normal or challenging. If there is a question about the ability of school-age children in deductive reasoning, then the same goes for student-teacher candidates. Prospective teacher students need logical reasoning when facing challenging problems or rather complicated problems, especially problem-solving questions because deductive reasoning is a high-level skill needed by students, especially prospective mathematics teacher students. Solving mathematical problems requires a deductive mindset which means that the process of doing mathematics is deductive. Mathematics accepts generalizations based on observation (inductive) but must be based on deductive proofs. Reasoning, the deduction is a thinking process that starts from existing proportions, leading to new propositions in the form of a conclusion.

(Ju & Choi, 2017), deductive reasoning is a process toward a special truth that is built from general truths. A reasoning that ends in a common event, that is, the truth is known and ends in a conclusion. So, deductive reasoning is a thought process that is general truth to special truth which ends with concluding. To find out how individual deductive reasoning can be seen from their ability to solve mathematical problems. Through the activities of solving mathematical problems, individuals can develop and build new ideas from existing knowledge. In solving math problems student teacher candidates will gain experience using the knowledge and skills they must apply to solve challenging problems.

A proposition is a statement in the form of a sentence that is judged to be true or false. A proposition is a statement that describes several conditions that are not necessarily true or false in the form of a sentence. The truth of a proposition corresponds to facts, a false proposition does not correspond to facts. In this study, the proportions used as the basis for conclusions are called general statements or special statements as well as the results of the conclusions. Deductive reasoning is a conclusion that departs from things that are general to specific things and is a conclusion that the process involves theories or other mathematical formulas that have previously been verified (Stylianides & Stylianides, 2013). Deductive reasoning is closely related to the process or activity of thinking to draw conclusions or make new statements by using or involving theories that have previously been proven true.

(Ayalon Michal and Even Ruhama, 2010), Deductive reasoning is a conclusion as an affirmation of what is already implied in the premise. This shows that the conclusion is a logical necessity of the premises and must be true if the premises are true. This means that if the premise is true, then the conclusion must also be true. Deductive reasoning is the truth of a concept or statement obtained as a logical consequence of the previous truth. (Lin & Guo li Taiwan shi fan da Xue., 2009) say that when someone reasons, someone uses prior knowledge about the truth of one or more statements to determine the truth of other statements. When someone reasoned the thing, he got a conclusion. In this case, to prove the truth of a statement, someone is doing the reasoning.

(El et al., 2008), states that problem-solving and proof are impossible without involving reasoning and both are ways in which students develop mathematical reasoning and understand mathematical ideas. In addition, evidence is a communication of reasoning that is built based on sense-making and is an important result of systematic thinking. This opinion explains that in mathematical proof, a person either consciously or unconsciously has used reasoning to be concluded.

(Cramer-Petersen & Ahmed-Kristensen, 2016) identifies deductive reasoning as i) making definite conclusions; ii) explaining the hypothesis and its reasons; iii) predicting conformity in each formulation; iv) proving something; v) knowing the consequences of the facts and evidence produced. Mathematical reasoning is known as axiomatic deductive reasoning, meaning that deductive reasoning is based on axioms or postulates. (Carreira et al., 2020), states the following aspects of deductive reasoning. i) explain the basic structure of the interrelationships between sets to find a solution to a problem; ii) recognize the logically equivalent formulation of a statement; iii) make decisions equivalent to identifying appropriate rules; iv) draw conclusions based on certain facts and rules. By looking at these aspects of deductive reasoning, it is only natural that deductive reasoning is needed by individuals when learning mathematics.

Mathematical reasoning is known as axiomatic deductive reasoning, meaning that deductive reasoning is based on axioms or postulates. (Rodrigues et al., 2021), define the reasoning process as i) generalization, namely identifying common problems and expanding reasoning beyond its original range; ii) justification, namely providing a logical sequence of statements based on the knowledge that is known to be true to make conclusions; iii) exemplifying, namely summarizing data from the problems encountered to produce elements that will be useful in generalizing and justifying; iv) compare, namely making conclusions by considering the similarities and differences of the statements given; v) classification, namely making statements between different objects based on common characteristic identities.

In this study, indicators of deductive reasoning by Carreira (2020)(Carreira et al., 2020) in solving number theory problems are presented in table 1. below.

Table 1. Indicators of Deductive Reasoning in Solving Number Theory Problems

Indicators	Aspects
Make general statements	Explain the basic structure of the relationship between the problem and the theory to find a solution to a problem
	Recognizes the logically equivalent formulation of a statement
Make Special Statements	Making decisions is equivalent to identifying the appropriate rules
Making Conclusions	Making decisions is equivalent to identifying the appropriate rules

(Shynkaruk, 2006) argue that deductive reasoning problems are analytical reasoning in which mathematical problems can be solved based on existing information, and the solutions obtained can be verified with normal logic. Deductive reasoning is much more complex than ordinary tasks. Usually, student-teacher candidates find it difficult to make conclusions. If it is not precise in recognizing the information provided, the problem will become more difficult.

Students who are future teachers of mathematics need to learn number theory because number theory is one of the foundations of mathematics whose universal set is integers. By studying number theory, students will be able to understand arithmetic well. In number theory, it discusses proving theorems which of course require deductive reasoning. For example, if students are asked to prove that 8 is divisible by $a^2 - b^2$ if a and b are two odd numbers, then with deductive reasoning student prospective teachers will give an example a = 2k +1 and b = (2k+1)+2t with k dan t integers, $a^2 - b^2 = (2k +1) - ((2k+1)+2t)^2 = -4t2 - 4t(2k+1)=-44t(t-2k-1)$ if t is an even number then $a^2 - b^2$ is divided by 8. Based on theorem 2.2, that is "if a is divisible by a number b, then a is divisible by m times b for every integer m". Therefore, if t is an odd number, then t-(2k+1) is an even number, so $a^2 - b^2$ is divisible by 8.

This study focuses on the deductive reasoning of prospective mathematics teacher students when solving number theory problems. This is done because previous studies have usually examined the deductive reasoning of prospective teacher students in abstract geometry or algebra material. Even though number theory is very important to be mastered by prospective mathematics teachers to deepen further mathematics material. Therefore, it is necessary to research the deductive reasoning of prospective mathematics teacher students in this number theory material so that they can describe how the reasoning of prospective teacher students. Thus, the results of this study can be used as a reference for designing learning theories so that learning outcomes can be optimal.

B. Methodology

This research was conducted on natural and developing objects as they are. Therefore, the researcher uses a qualitative descriptive research method to describe the deductive reasoning of mathematics teacher candidates. The study was designed to give tests to two research subjects on deductive reasoning problems and to be interviewed to obtain in-depth data. The subjects in this study had the same mathematical abilities as seen from the results of the final exams at the end of the first semester and were female. The problem of deductive reasoning given to research subjects is as follows.

> The greatest common factor of two numbers is one. When added, the two numbers are divisible by an integer. What is the greatest common factor of the two numbers and their divisor?

Figure 1. The Problem of Deductive Reasoning

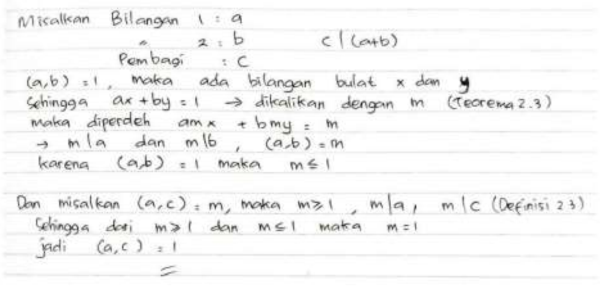
The interview used was a semi-structured interview which was conducted after the subject worked on deductive reasoning problems. The results of deductive reasoning tests and interviews and the analysis were carried out based on indicators of deductive reasoning, namely by summarizing and selecting the things needed in presenting and making conclusions about the deductive reasoning of student-teacher candidates.

C. Findings and Discussion

Findings

Descriptive Analysis

Researchers chose two research subjects, namely HNW and MAF. Data analysis in this study was carried out based on indicators of deductive reasoning. The figure 2 was the answers from the HNW and MAF subjects.


Misalkan Bilangan 1 : a
2 : b c | (a+b)
Pembagi : c
(a,b) = 1, maka ada bilangan bulat x dan y
Sehingga ax + by = 1 → dikalikan dengan m (Teorema 2.3)
maka diperoleh amx + bmy = m
→ m | a dan m | b , (a,b) = m
karena (a,b) = 1 maka m ≤ 1
Dan misalkan (a,c) = m, maka m ≥ 1 , m | a, m | c (Definisi 2.3)
Sehingga dari m ≥ 1 dan m ≤ 1 maka m = 1
jadi (a,c) = 1

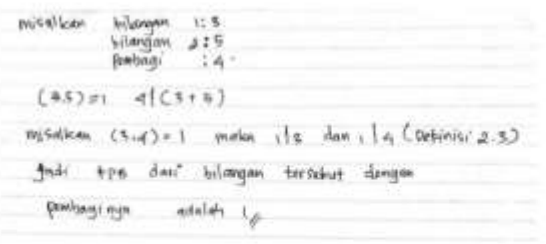
Translate Version:
Let: first number : a
second number: b c | (a,b)
divider : c
(a,b) = 1, then there are integers x and y
So that ax + by = 1 ⟶ multiplied by m (theorem 2.3)
Then obtained amx + bmy = m
⟶ m | a dan m|b, (a,b) = m
Because (a,b) = 1 then m ≤ 1
And let (a,c) = m, then m ≥ 1, m | a, m |c (definition 2.3)
So, from m ≥ 1 and m ≥ 1 then m = 1
So (a,c) = 1

Figure 2. HNW Subject Answers

The first indicator, namely making general statements with aspects of understanding the basic structure of interrelationships between sets to find a solution to a problem. Based on Figure 1. it can be seen that the HNW subjects understand the basic structure of the interrelationships between sets to find solutions to the problems given. The HNW subject wrote an example of numbers and "Theorem 2.3", this indicated that the subject understood the relationship between the problem and the theorem that had been studied. Likewise, in the results of the researcher's interview with the subject HNW, the subject said that "This problem has something to do with the theorem that has been studied, namely Theorem 2.3". The first indicator is on aspects, namely recognizing logically equivalent formulations of a statement. The HNW subject wrote "(a,b)=1, then there are integers x and y so that ax+by=1", this implies that the subject recognizes logically equivalent formulations. The HNW subject said, "I multiplied a and b by x and y and the result is equal to 1 according to theorem 2.3".

The second indicator, namely making specific statements with aspects of making decisions that are equivalent to identifying appropriate rules. In Figure 1. It can be seen that the HNW subject writes the word "then" which is associated with theorem 2.3 and definition 2.3. This implies that the subject has decided according to the rules. Likewise, with the results of the researcher's interview with the subject HNW "Based on theorem 2.3 and definition 2.3, I know that m = 1".

The third indicator with the aspect of making conclusions based on certain facts and rules. In Figure 1. It can be seen that the HNW subject has made conclusions according to the facts by writing the word "so" after analyzing the facts by writing "so that from $m \geq 1$ and $m \leq 1$ then equation when with the results of the researcher's interview with the HNW subject "when viewed from m = 1, then (a, c) = 1". This implies that the greatest common factor of two numbers and their divisor is one.



Translate Version
Let: first number : 3
second number : 5
divider : 4
(3,5) = 1, 4 | (3+5)
Let (3,4) = 1 then 1 |3 and 1 | 4 (Definition 2.3)
So The greatest common divisor of the number and its divisor is 1

Figure 3. MAF Subject Answers

The first indicator on the aspect explains the basic structure of the relationship between the problem and the theory to find a solution to a problem. MAF subjects understand the interrelationships between sets even though they don't give examples of numbers, but through examples, they write "Definition 2.3" (Figure 3). The subject said "This can see in definition 2.3" during an interview with the researcher. This implies that the subject understands the relationship between the questions and the definitions that have been studied. While the first indicator on the aspect recognizes a logically equivalent formulation of a statement, the MAF subject implicitly fulfills the second indicator by writing "for example (3,4) = 1 then 1|3 and 1|4 (definition 2.3)" and based on the results of interviews with researchers "according to definition 2.3 that if the common factor is the greatest of two numbers is one, then these numbers are divisible by one."

The second indicator of aspects of making decisions is equivalent to identifying appropriate rules. MAF subjects also make decisions by the equivalent rules, namely by writing the word "then" and then linking it to definition 2.3. The same thing is implied from the results of the researcher's interview with the subject of MAF "The divisor is one, I get based on the definition of 2.3".

The third aspect indicator makes conclusions based on certain facts and rules. MAF subjects have also written the word "so" in making conclusions that are by the facts, namely based on definition 2.3. This is to the results of interviews between researchers and MAF subjects "I believe the greatest common factor of these numbers is one based on the definition of 2.3".

Discussion

Based on the analysis of the data, the prospective teacher students in this study fulfilled the indicators of deductive reasoning that had been determined by the researchers, even though there were differences in how to analyze them. The first prospective teacher students work with something general and relate it to theorems and definitions. This is by Lithner (2000(LITHNER JOHAN, 2000)) who states that deductive reasoning is a process of reasoning from general knowledge of principles or experience that leads us to conclude something special. On the other hand, (Cramer-Petersen & Ahmed-Kristensen, 2016) state that the mathematics teacher candidate must learn how to justify a statement that exists at three levels: doing the proof, understanding the nature of the proof, and adapting the proof of concept to different levels of development.

Meanwhile, the second student-teacher candidate works by exemplifying numbers, or it can be said that the student-teacher candidate works with something special, then relates it to the definition. This implies that prospective teacher students solve problems by using examples although, in the end, they use general things to conclusions. What these student-teacher candidates do is (El et al., 2008) statement, it can be pointed out that there are still many students at the first level of tertiary institutions who think in the concrete operational stage with inductive reasoning. There are still many students who are less able to learn mathematics by using a deductive mindset. The deductive mindset is simply said to be thinking that stems from things that are general and brought to specific things.

In addition to the things that have been disclosed by the researcher, the researcher assumes that other influences cause differences in the way prospective teacher students work on the given deductive reasoning questions, namely individual characteristics related to how to process information, store, solve problems, and how to make decisions. Of course, many factors influence it. The way individuals obtain information and process it is usually done consistently. Some individuals are global, namely, individuals who accept something globally and have difficulty separating themselves from their surroundings or are more influenced by the environment. Individuals with such characteristics are called individuals with field-dependent cognitive styles. On the other hand, some individuals are analytic, that is, they tend to describe the background of existing problems and can distinguish objects from the surrounding context and view their surroundings more analytically so that individuals who are like this are not easily influenced by the surrounding environment. Individuals with these characteristics are called individuals with field-independent cognitive styles. (Witkin & Goodenough, 1977) states that analytic individuals are individuals who separate the environment into its components, are less dependent on the environment or are less influenced by the environment. While global individuals are individuals who focus on the environment as a whole or are influenced by the environment.

Therefore, researchers provide cognitive style tests to determine individual characteristics in obtaining, storing, and processing information. This test is called the Group Embedded Figures Test (GEFT). The test results stated that the first student-teacher candidates (HNW subjects) had field-independent or analytic cognitive styles, individuals who separated the environment into its components or were less influenced by the environment. Meanwhile, the second student teacher candidate (MAF subject) has a field-dependent cognitive style, and is global or influenced by the environment. Thus, it can be said that individual deductive reasoning can also be influenced by individual characteristics in processing information or cognitive style. This is because everyone has unique reasoning characteristics, which are not shared by other individuals.

D. Conclusion

The results of this study indicate that second-semester mathematics teacher candidates reason deductively through four components, namely: explaining the basic structure, recognizing equivalent formulations, making equivalent decisions by identifying appropriate rules, and making conclusions based on facts and rules. The deductive reasoning of prospective teacher students can also be influenced by cognitive styles. In addition, the deductive reasoning of future mathematics teacher-students is a potential tool for developing logical competence in learning mathematics which will later be useful when practicing in-school mathematics learning.

Therefore, researchers suggest the importance of developing deductive reasoning for prospective mathematics teacher students so that they can easily practice it in everyday life, especially when they have entered school. In addition, to designing learning, teachers should also pay attention to individual characteristics in receiving, storing, processing information, and how these individuals solve problems so that learning objectives can be achieved.

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