Promoting Self-Reflection Over Re-Teaching: Addressing Students’ Misconceptions With ‘My Favorite No’

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ARTICLE INFO
o-ISSN: 2528-2026
p-ISSN: 2528-2468
Vol. 5, No. 2, December 2020
URL: http://doi.org/10.31327/jme.v5i2.1230

Suggestion for the citation and bibliography
Citation in text:
Caniglia (2020)

Bibliography:

Abstract
Misconceptions are an important aspect of learning and teaching mathematics. What are effective ways for teachers to confront misconceptions and prevent their reoccurrence? This mixed-methods study examined instrumental and conceptual understandings of students’ errors, mistakes, and misconceptions in a 7th grade mathematics classroom utilizing the ‘My Favorite No’ strategy. Fifty-five students from a Midwest middle school were divided into two groups with similar abilities, with one group confronting misconceptions using the ‘My Favorite No’ strategy while the other group received information about a potential error directly from a teacher. A brief survey of questions followed for students in the ‘My Favorite No’ class. One group was concerned with conceptual understanding and making connections, and able to solve similar problems. In the second group, students improved from pretests, yet did not improve as much as students using ‘My Favorite No.’ Thus, teachers are encouraged to consider using ‘My Favorite No’ a strategy where students are in an environment in which they discover their errors and believe in the value of making a mistake.

Keywords: conceptual understanding, instrumental understanding, misconceptions

A. Introduction
The role of misconceptions, arguably, plays a significant role in learning mathematics. For this reason, “an understanding of student misconceptions, and effective strategies to help students avoid them, is an important aspect of mathematical pedagogical content knowledge” (Chic & Baker, 2005). Teachers of mathematics understand this and try to find ways to help students avoid and overcome common misconceptions that are associated with specific content standards. This begs the question, "What are effective ways for teachers to confront misconceptions and prevent their reoccurrence?"
My Favorite No is an informal assessment technique inspired by the Teaching Channel, in which students’ mistakes are turned into collective opportunities for learning. The technique involves a teacher giving students an opportunity to answer a question and then analyze a wrong answer given by a classmate by following three simple steps: 1) students write answers to a question and teacher collects the responses, 2) teacher, together with the students, sort the answers into "yes" and "no" and 3) from the "no" responses, the teacher selects a favorite no response and asks students to identify what is wrong with the response (Roach, 2014). The name of the technique, My Favorite No is derived from the teacher’s ‘favorite’ incorrect response selected for analysis. This activity allows the teacher to quickly assess how students are grasping the concept and for those who are not, what is causing their misunderstanding. The rationale for this strategy is that “formative assessment can enhance learning when designed to provide students with feedback about particular qualities of their work and guidance on what they can do to improve” (National Research Council, 2001). Therefore, if teachers can expose the students’ mistakes in class, this can enhance learning. This was the basis for this study, to establish if My Favorite No is an effective approach for teachers to confront misconceptions and prevent their reoccurrence?

This article follows three lines of inquiry (a) contributions of misconceptions to the process of learning mathematics; (b) literature that demonstrates misconceptions do not occur randomly but originate within a conceptual framework based on previously acquired knowledge; and (c) a recommended change in perspective from one that decries errors and purposely allows for them in the process of learning. The authors utilized these research lines as they investigated the efficacy of one strategy known as My Favorite No.

B. Literature Review

1. Contributions of Misconceptions to Learning

In his book Conjectures and Refutations, Karl Popper provides arguments on truths in science. He claims that "Erroneous beliefs may have an astonishing power to survive, for thousands of years" (Popper, 1963), and although he does not give a method that would lead us to see the truth in all circumstances, he suggests reversing the question about "sources of our knowledge" into - "How can we hope to detect error?". According to Popper then, if one can detect an error, one is in a better place to improve upon his or her beliefs. If we adopt a "critical search for error" our previous ways of thinking may change and become more powerful.

The following citation from Nesher (1987) is consistent with Popper’s philosophical treaties on truth and can be applied to learning and teaching: Is all this relevant to the child’s learning? I believe it is. If I replace the terms ‘true and false’ with ‘right and wrong’ or ‘correct and erroneous’ we will find ourselves in the realm of schools and instruction, where, unlike in the philosophical realm, "being wrong" and "making errors" are negatively connotated. The system, in fact, reinforces only "right" and "correct" performances and punishes "being wrong" and "making errors" by means of exams, marks, etc., a central motive in our educational system.

Piaget’s (1972) understanding of cognitive dissonance is also relevant to a discussion on student misconceptions. In his theory of the construction of knowledge, he suggested that mental structures or schema are constructed through interaction by processes called assimilation and accommodation. Assimilation occurs when new ideas are added to what a person already knows (existing schema) while accommodation occur when the schema needs to be restructured to make room for new information (Piaget, 1972). Once a schema or concept is formed, it is fairly resistant to change. Errors in computation are not significant unless the errors or misconceptions are diagnosed and addressed. It is a significant part of the learning process if these errors are dealt with diagnostically. Misconceptions and errors must not be seen as obstacles or ‘dead ends’ but must be regarded as an opportunity for reflection.

2. Origins of Mathematical Misconceptions

Skemp’s (1977) distinction between relational and instrumental understanding suggests the types of understanding necessary for addressing misconceptions while informing their origins. According to Skemp, relational understanding is explained as an understanding of knowing how a generalization was reached so that one may reconstruct the problem-solving process at a later time. An instrumental understanding of mathematics is deductive and refers to remembering a
formula and its application, with little appreciation as to how it was derived (Skemp, 1987). For example, one can know that the formula to calculate the area of a triangle is $A = \frac{1}{2} \text{base} \times \text{height}$ without knowing how it was derived. Such understanding only enables one to use the formula to get correct answers and ultimately claim that one understands how to determine areas of triangles, whereas in actuality, they may not understand because if they forget the formula, they will not be able to work out the problem (Sarwadi & Shahrrill, 2014). From this perspective, instrumental understanding may be the basis of many mistakes in the work that involves mathematical tasks involving the application of rules and algorithms.

Mistakes, errors, and misconceptions all indicate that the solution of a mathematical task is not correct, yet there are distinctions in the terms throughout the literature (Erwlanger, 1975). All three can be placed on a continuum with one end including slips or mistakes while at the opposite end lie misconceptions. Mistakes or errors can result from misreads or instantaneous lapses in memory such as misreading a sign or adding digits incorrectly (Erwlanger, 1975). On the opposite end lie misconceptions or systemic errors which are more serious. Systemic errors are identified by recurrent wrong answers methodically re-applied in answering particular types of mathematical questions. Nesher (1986) explains that “the notion of misconception denotes a line of thinking that causes errors, all resulting from an incorrect underlying premise, rather than sporadic, unconnected or non-systematic errors”.

3. A Change in Perspective

Brain plasticity research demonstrates that the brain has an ability to grow new connections over time, and this idea has brought the growth-mindset revolution into the world of education. “When students think about why something is wrong, new synaptic connections are sparked that cause the brain to grow” (Boaler, 2013). Therefore, warning students about common misconceptions may not provide them with the ability for increased performance as it does not create an opportunity to learn from their mistakes. This lack of opportunity denies students the chance to self-reflect and analyze their thoughts.

Metacognition, or thinking about your own thinking, can help students identify their strengths, weaknesses, and errors in thinking. Like many skills, this is a process that develops over time. “Metalearning skills can be improved through practice, guidance, and encouragement” (Henderson & Harper, 2009). Constantly reminding students of common misconceptions prevents them from developing their metacognition abilities. One such activity, My Favorite No, is a strategy that is described as a promising technique to acknowledge and correct student misconceptions.

In this activity, inspired by The Teaching Channel (2014) and Rossman and Chance’s What Went Wrong (2004), students will answer a question provided by their teacher and then analyze a wrong answer given by a classmate. The purpose of this activity is for the teacher to quickly assess how many students are grasping the concept and for those who are not, what in particular is causing their misunderstanding. It is essentially a formative assessment that works particularly well as a warm-up to start a class. It is imperative that enough time be allotted for the analysis of the wrong answer. It can work in all content areas and across grade levels. Implementations consists in the following steps:

1. Ask students to solve a math problem on an index card, then turn it in to you
2. Sort the cards into piles for correct and incorrect answers. Take a moment to find an especially good mistake – one that lots of students make or one that highlights an important math concept. Recopy the incorrect answer to a new card, so that student handwriting cannot be recognized
3. Ask students to identify what was done well. Then ask them to find where the mistake occurred. Have them explain and justify their thinking.

Rossman and Chance (2004) supports similar activities in statistics education because of the importance of recognizing student misconceptions, “the primary goal is to maintain a constructivist approach by presenting students with sample responses that contain some kind of error and asking them to identify and correct the error”.
C. Methodology

1. Participants

This study occurred in a suburban middle school located in the Midwest. The primary author was the teacher of two seventh-grade classrooms. One classroom (using My Favorite No) was comprised of 28 students, fourteen of whom were girls, and fourteen of whom were boys. The second classroom (using direct instruction) was comprised of 27 students, 14 of whom were girls, and 13 of whom were boys. Mathematics classrooms were not tracked and did not include inclusion specialists—both classes were similar in mathematical ability. Although all work was required, if students or parents did not consent, their responses were not included within the results. All students and parents agreed to participate.

2. Research Design and Analysis

Data were collected on five separate occasions within each of the two mathematics classes over a four-month time period. During these class periods the data were collected from five sessions where the teacher utilized My Favorite No activities with one class, and with another class of similar demographics discussed student misconceptions in a direct teaching format. Both classes were surveyed at the end of the study to gather their opinions on the two different teaching techniques.

For each session, both classes either completed a quiz or math test that assessed topics that were covered in a particular unit in their math class. These quizzes and tests were analyzed before the teacher selected the problems for the My Favorite No activity. After the teacher addressed the misconceptions found from the class using My Favorite No, the same misconceptions were taught to the direct instruction class. Following this instruction, a formative post assessment was administered to both classes within a week. The post assessment contained the same type of problems as the one selected for My Favorite No, to see if the potential knowledge gained was transferable. In this case, transferable refers to the student’s ability to solve similar types of problems.

The independent variable in this study was the teaching strategy used to address the students’ misconceptions (My Favorite No vs. Direct Instruction), and the dependent variable included the post assessment results associated with each class. Other than the way the teacher addressed the misconceptions with the class, the same lesson plan was used for both classes in an attempt to control for other variables. This study examined the effects of the My Favorite No strategy on students’ abilities to learn from their misconceptions by comparing pre- and post-test results on the mathematical concept that was being addressed.

At the end of the study, the students who participated in the My Favorite No assessment were surveyed, and were asked the following questions:

- What do you think is a better way to prevent a mistake in the future? (Your choices are participating in My Favorite No or having a teacher talk to you about the mistakes themselves).
- How would you feel (or how did you feel) if your teacher chose your mistake during My Favorite No?
- What are your feelings about making a mistake in mathematics after doing My Favorite No?

D. Findings and Discussion

1. Findings

For the first data collection point, the question selected for My Favorite No required the students to convert 125% into an equivalent fraction. Seven students thought that the equivalent fraction for this percentile was 125/1000. When this error was presented to the class during My Favorite No, the students decided that the misconception shared among the students was that the thousandths place was three spaces to the right of the decimal point, and 125% takes up three spaces to the left of the decimal point. After identifying a common misconception, this concept was taught directly to the second class.

The post assessment was given to both classes five instructional days after the one class completed their first My Favorite No. This assessment required the students to write 117% as an equivalent fraction and as a decimal. The students were asked to write the percent as a decimal to see if any knowledge they had gained would be transferable to the concept of percent/decimal
conversions. The post-test results for the students who were selected for *My Favorite No* can be viewed in Figure 1.

![Figure 1](image1)

**Figure 1**: *Data collection 1.* This figure illustrates the data collection from the first *My Favorite No* activity.

As seen in Figure 1, both classes scored a 0% on the pre test, although the *My Favorite No* class scored 10% higher on the post-test than did the direct teaching class, and were able to transfer their knowledge of fractions to writing decimals 40% more than the class that received direct instruction. This impressive result may have been due to the fact that this was the first time that the students have engaged with a new instructional strategy.

For the second data collection point, the question selected for *My Favorite No* required the students to solve the problem $6 + | -8 + 3 |$. Nineteen students arrived at the solution of 17. When this error was presented to the class using *My Favorite No*, the students decided that the misconception shared among the class was the order in which the absolute value was used. For example, students found the absolute value of 8 and 3 separately and then added them together. During the discussion, one student shared that the students who missed the problem should think of the brackets as parenthesis and add the two numbers together before finding the absolute value of their sum.

After the second *My Favorite No* activity and direct instruction based upon the misconception, a post assessment was given four days later to both classes. This assessment required the students to solve the problems $7 + | -9 + 5 |$ and $-6 + | 6 + -3 |$. The post-test results for the students who were selected for *My Favorite No* can be viewed within Figure 2. In this case the direct instruction class scored 10% higher although both classes showed large gains from the pre-to post-tests.

![Figure 2](image2)

**Figure 2**: *Data collection 2.* This figure illustrates the data from the post-test of *My Favorite No*.

For the third data collection, the question selected for *My Favorite No* required the students to solve the problem $2(-1.5)$. Eight students thought that the answer was 0.5. When this error was
presented to the class during *My Favorite No*, a student pointed out that the students who reached an answer of 0.5 thought that you had to add the numbers instead of multiplying them.

The post assessment was given five days after *My Favorite No* was completed. This assessment required the students to solve the problems 3(-2.5) and \( \frac{1}{2} \) (%). The students were also asked to solve 6.4 + 4.3(-2) and 3(-3/5) + 10 × 2 to see if any knowledge they had gained from *My Favorite No* would be transferable to the order of operations. The post-test for the students who were selected for *My Favorite No* can be viewed within Figure 3. Within Figure 3, the students in the *My Favorite No* class showed a 13% increase on the post-test as compared to the direct instruction class and also were able to transfer their knowledge to similar problems 44% more than the other class.

![Data Collection 3](image1.png)

**Figure 3.** Data collection 3. This figure illustrates the percentage of transferable knowledge from *My Favorite No*.

For the fourth data collection, the question selected for *My Favorite No* required the students to solve the problem \((-1) + (-1) - (-9) + (-1)\). Eight students thought that the answer was 8. When this error was presented to the class during *My Favorite No*, a student pointed out that the students thought while performing the order of operations you should always do adding instead of subtracting. It was then explained by another student that they are the same step, and that you should work from left to right when addition and subtraction remain.

The post assessment was given seven days after *My Favorite No* was completed. This assessment required the students to solve the problems \((-2) + (-2) - (-8) + (-1)\) and \((-3) + 4 - (-7) + 5\). The students were asked to solve \((-5) + (-3) - (-4) + 3(-2)\) and \((-2) \times (-9) + 6 \times (-3)\) to see if the knowledge they had gained from *My Favorite No* was transferable to other order of operations concepts. The post-test results for the students who were selected for *My Favorite No* can be viewed within Figure 4. Within Figure 4, the students in the *My Favorite No* class scored 48% higher on the post-test and 35% higher when solving a similar problem.

![Data Collection 4](image2.png)

**Figure 4:** This figure illustrates the post-tests compared among the direct teaching and *My Favorite No* class.
For the fifth data collection point, the question selected for *My Favorite No* required the students to convert 9/8 into an equivalent percent. Six students thought that the answer was 11.25%. When this error was presented to the class doing *My Favorite No*, a student pointed out that the students might have thought that you cannot go over 100%, and so they moved the decimal point two spots to the left of the last digit. The post assessment was given eight days after *My Favorite No* was completed. This assessment required the students to convert 6/4 and 7/3 into equivalent percentages. The students were asked to convert 125% to an equivalent decimal to see if the knowledge they had gained from *My Favorite No* was transferable to percent/decimal conversions. The post-test results for the students who were selected for *My Favorite No* can be viewed within Figure 5. The students who participated in *My Favorite No* showed a growth of 85.1% on the questions they originally missed, while the class who received direct instruction to address their misconceptions showed an average growth of 76.6% on the same questions. Regarding the questions meant to see if the knowledge from *My Favorite No* was transferable to other mathematical concepts, the students who participated in *My Favorite No* were able to solve 82.6% of the questions. The students who received direct instruction to address their misconceptions were able to solve 43% of the questions designed to test transferability.

![Data Collection 5](image)

**Figure 5**: *Data collection 5*. This figure illustrates the transferability of post-test

After completing the survey, 82% of the students who participated in *My Favorite No* said that they prefer that strategy over direct teaching knowledge among classes. When the students were asked, which technique was better to prevent mistakes in the future, some of them commented:

- *"My Favorite No" is better because it is easier to learn from mistakes if you actually have to think about it and why it’s wrong."*
- *"My Favorite No" is better because we learn that mistakes are more of a positive than a negative."*

When the students were asked how they would feel if a teacher confronted them with their mistake during class, some of them replied:

- *"I wouldn’t care because it’s constructive criticism."*
- *"Fine because then I know not to make that mistake anymore, and neither does the class."*

When the students were asked about their feelings regarding mistakes after participating in *My Favorite No*, some of them said:

- *"I think they accelerate your thinking and you get to improve the skills you have."*
- *"We all make mistakes, and we’re all like a big team."*
- *"I think that making mistakes can make you get better."*

2. **Discussion**

The authors concur with Van de Walle (2012) who found that “enhanced attention to mistakes improves performance after the error”. In this study, one class was required to reflect on their errors by participating in *My Favorite No*, and the other class received direct instruction from their teacher that was designed to prevent them from making the same mistakes in the
future. During one part of the study, students appeared to benefit from both approaches (see Figure 2). However, the class that was required to reflect on their mistakes showed growth according to their post-test results (see Figures 1, 3, 4, and 5).

The most helpful benefit of My Favorite No was associated with its ability to help students transfer the knowledge they gained from the activity to similar mathematical problems. The students who participated in My Favorite No overall scored higher on the questions regarding transferability. It was especially interesting to compare the transferability results from the first and fifth sessions because they both required the students to convert a percentage into a decimal as students who participated in My Favorite No scored higher than the class receiving direct instruction in both instances.

As the students continually participated in this activity, they began to develop a growth-mindset. Boaler (2013) suggests that “mistakes should be valued for the opportunities they provide for brain development and learning”. The learning process becomes more engaging for the students when they understand that they are in control of their cognitive development. My Favorite No provided added benefits to the students having their mistakes presented to the class. Bem (1967) found that “if a person holds two cognitions that are inconsistent with one another, they will experience the pressure of an aversive motivational state called cognitive dissonance”. When a student’s error is presented to the class in this format, they are able to reflect on their thinking as well as hear what others have to say about approaching the problem. This dissonance forces students to wrestle with their own thoughts and ideas while also making them want to identify their mistakes.

While this study found My Favorite No to be quite beneficial in terms of students learning from their misconceptions, it is clear that there are important factors that teachers should consider in order for this strategy to reach its true potential. Stiggins and Chappius (2005) asserts that teachers “must deliver assessment results into the hands of their intended users in a timely, understandable, and helpful matter”. If a teacher waited too long to recognize misconceptions it would be harder for students to reflect on their thought process. Educators practicing this instructional strategy need to put it into action in a timely fashion for the students to realize the true benefits it has to offer.

While choosing a student work sample to present to the class for My Favorite No, teachers need to be able to select problems that will cause cognitive dissonance. “A careless error has been defined as one in which occurred even though the student knew how to gain a correct answer to the question at the time the incorrect answer was given and would be expected to give the correct answer when responding to the same question at some later time” (White, 2005). It’s imperative for a teacher to be able to differentiate between a careless error and a misconception when selecting a problem to present My Favorite No. Identifying an error in a careless mistake does not cause a student to think nearly as much as when they have to try to figure out conceptual misunderstandings.

E. Conclusion

This study was designed to shed light on how to address students’ misconceptions in order to improve their achievement in mathematics. The nature of students’ misconceptions and how to address them has long been a topic of interest of mathematics educators. Math teachers will often analyze students’ work to try to understand their thought process when they notice an error. As this study showed, analyzing students’ thinking and identifying their misconceptions is important, but what educators do with this information is what’s really crucial in terms of helping their pupils develop conceptual understanding.

My Favorite No is a meaningful, engaging activity that is designed to make students reflect on their peers’ and their own thinking in an attempt to identify mathematical misconceptions. Requiring students to analyze errors is not the only benefit associated with this activity. My Favorite No helps students understand that mistakes are a valuable part of the learning process and motivates students to view learning as a deeper experience. This strategy has the potential to allow students to engage in self-reflection which may lessen their dependence on external rewards such as grades. Zimmerman (2011) found that students who practiced self-reflection did not view “the reception of an academic grade as an end-point of learning, they learned to view it as an opportunity for further learning”. 
The results of this study were very promising in terms of showing the effectiveness of *My Favorite No* as a strategy to help students address their misconceptions and transfer their knowledge to other mathematical concepts. It engages students in active learning where they are analyzing errors instead of being told how to fix them. It requires problem-solving and self-reflection, and makes students look at mistakes as opportunities for growth, not as a sign that they can't solve problems. Teachers who are willing to execute this strategy regularly in their classroom are likely to see large improvements in their students’ mathematical understanding.

Implementing the strategy in the classroom as bell work or exit ticket is easy. According to Roach (2014), the strategy can be done with any math topic or content area, takes very little time, and it can be woven into the daily routine of class by following the three easy steps. As a formative assessment approach, it works particularly well when used as a warm-up or do now activity to start a class. To ensure success of this approach, it should be noted that is imperative that enough time be allotted for the analysis of the wrong answer. It can work in all content areas and across grade levels.

**F. References**


